GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Posted ack'd OHL

Date: 8/10/78

Project Title: National Wind Energy Statistics for Large Arrays of Wind Turbines

Project No: E-16-617

Project Director: Dr. C.G. Justus

Sponsor: U. S. Dept. of Energy; Richland Operations Office; Richland, Washington, 99352

Effective Termination Date: 4/30/78 (Mod. A001 period)

Clearance of Accounting Charges: by 4/30/78

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- x Final Fiscal Report (Cert. Stmt. of Costs per Appendix "C") due by 7/31/78.
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______

NOTE: Follow-on project is G-35-641

Assigned to: Aerospace Engineering (School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
Dean/Director—EES
Accounting Office
Procurement Office
Security Coordinator (OCA)
Reports Coordinator (OCA)

Library, Technical Reports Section
Office of Computing Services
Director, Physical Plant
EES Information Office
Project File (OCA)
Project Code (GTRI)
Other ______

CA—4 (3/76)

Energy Statistics for Large Wind Turbine Arrays

Monthly Administrative Report for the

Period: 1 May 1977 - 31 May 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

July 8, 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

DIVISION OF SOLAR ENERGY

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - Because paperwork initiating contract renewal was not received until the first of July no major work could be cone on the contract. In preparation for the work necessary data tapes were ordered.

2. Financial and Administrative Status - Because of the delay in contract renewal initiation, no financial charges or manpower commitments against the project were made for the month. Cost, Manpower, and Milestone Status are shown on the attached form.

Respectfully submitted:

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

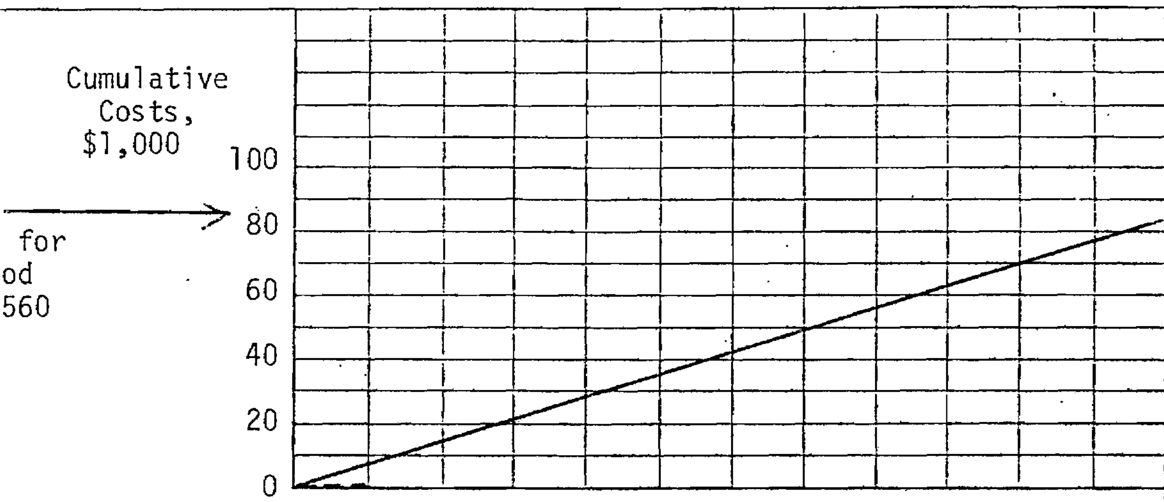
FORM APPROVED OMB NO. 38R0

1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: 5/1 through 5/31	3. Contract Number: EY-76-5-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

7. Months: M | J | J | A | S | O | N | D | J | F | M | A — 8. FY 1978

9. Cost Status

a.



Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	6.9											
c. Actual	$K	0											
d. Variance	$K	6.9											

e. Actual Costs Prior FYs: $82,047

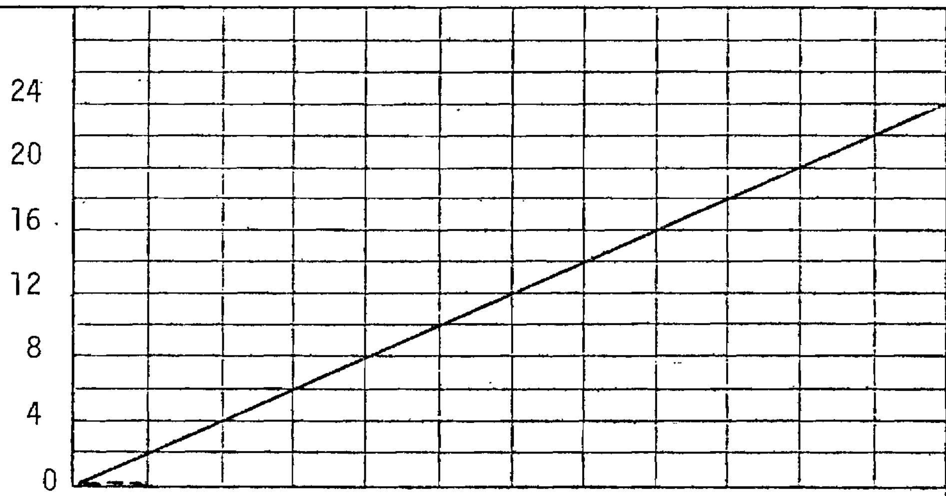
f. Planned Costs Prior FYs: $82,047

g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

10. Manpower Status

a.



Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	2											
c. Actual	0											
d. Variance	2											

e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

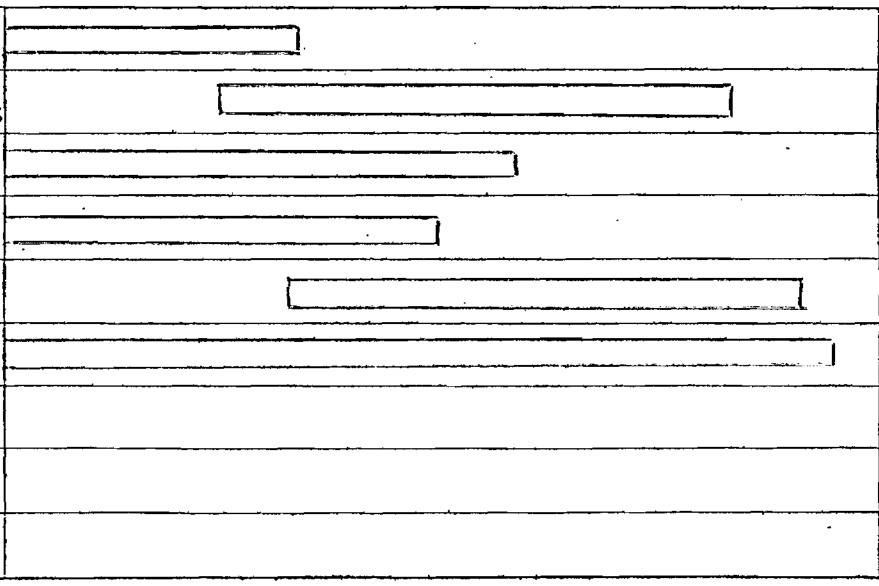
d. Array w/Storage Prog.

e. Storage Analysis

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date: 7/8/77

14. Signature of Government Technical Representative and Date

Energy Statistics for Large Wind Turbine Arrays

Monthly Administrative Report for the

Period: 1 June 1977 - 30 June 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

July 8, 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - Because paperwork initiating contract renewal was not received until the first of July no major work could be done on the contract. In preparation for the work necessary data tapes were ordered and have been received. With all programs set up to run on these data it is anticipated that we will be able to get back on schedule, despite the initiation delays.

2. Financial and Administrative Status - Because of the delay in contract renewal initiations, no financial charges or manpower commitments against the project were made for the month. Cost, Manpower, and Milestone Status are shown on the attached form.

Respectfully submitted:

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

FORM APPROVED
OMB NO. 38-R-

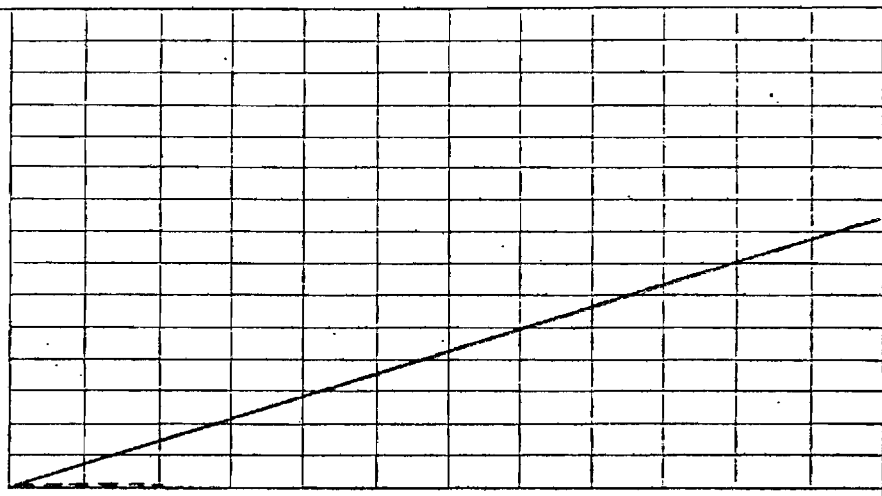
1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: ~~6/1/~~ through 6/30	3. Contract Number: EY-76-5-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

7. Months: M J J A S O N D J F M A — 8. FY 1978

9. Cost Status

a. Cumulative Costs, $1,000

Total Period $83,560



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

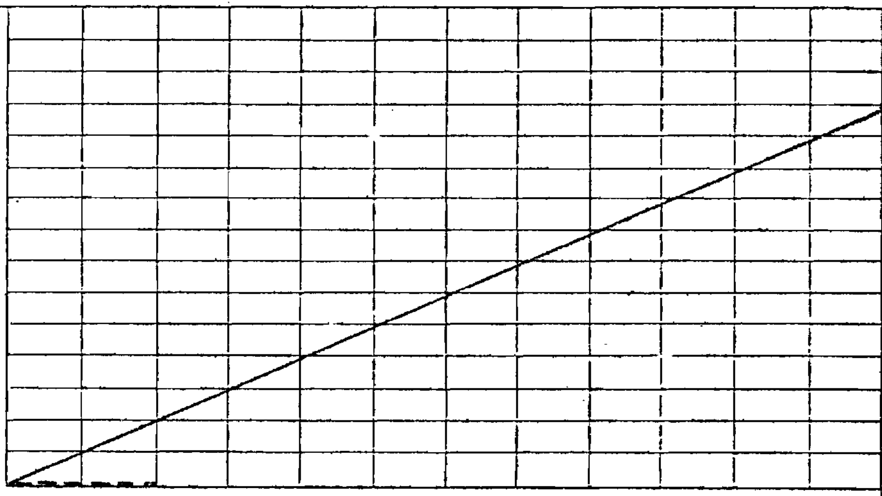
g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	6.9	6.9										
c. Actual	$K	0	0										
d. Variance	$K	6.0	6.9										

10. Manpower Status

a. Cumulative Man Months



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned		2										
c. Actual		0										
d. Variance		2										

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

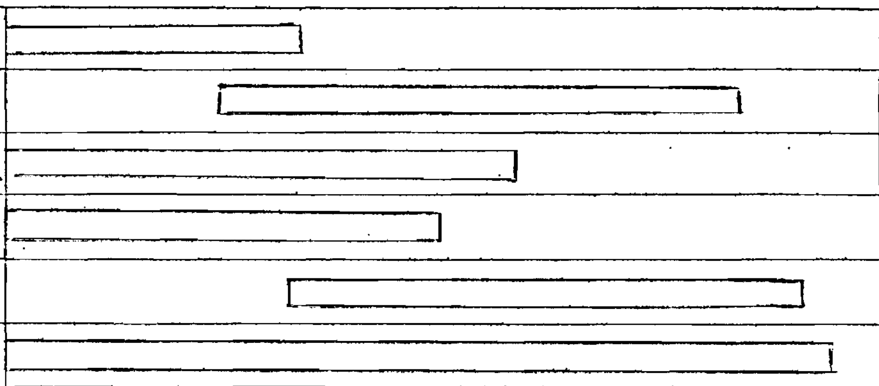
d. Array w/Storage Prog.

e. Storage Analysis

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date

14. Signature of Government Technical Representative and Date

E-16-617

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 August 1977 - 31 August 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

September 8, 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - Because paperwork initiating contract renewal was not received until the first of July, technical progress is still behind schedule, although catching up. Sorting of the multi-region data files is finished for 1971 data but minor problems were noted for two or three sites. These are being corrected before proceeding to sort other years. The 1971 data have been run through the array analysis program and, except for the sites with data problems, everything went well. Therefore analysis of the full data set should proceed quickly as soon as data sorting is complete.

   The array with storage program has been written and checkout runs are ready to be done. In conversations with the project monitor, the possibility was discussed of discontinuing this task and substituting another task area, because of the apparent lack of interest in storage with wind systems. Some possible substitute tasks were discussed and a decision will be made in late September.

   Problems with the data logger for the wind turbine test facility were noted and it was sent back to the manufacturer for repair. Complete installation of the data logging and recording instrumentation awaits return of the data logger. The wind turbine is operational and early test runs have been done. Power output data is awaiting periods of stronger winds, which are expected in the fall seasons, the summer winds in Atlanta are geneally light and variable.

2. Financial and Administrative Status - Because of the delay in contract renewal initiation, project manpower and expenditures are still running behind planned levels. Additional manpower is being devoted to the project until it is back on schedule. Encumberances of $7,600 for the full equipment purchases during the next month will also bring the cumulative expenditures essentially up to planned levels.

Respectfully submitted:

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

FORM APPROVED OMB NO. 38-R-

1. Contract Identification	2. Reporting Period	3. Contract Number
Energy Statistics for Large Wind Turbine Arrays	~~8/1~~ through 8/31	EY-76-5-06-2439

4. Contractor (name and address)
School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332

5. Contract Start Date: 5/1/77

6. Contract Completion Date: 4/30/78

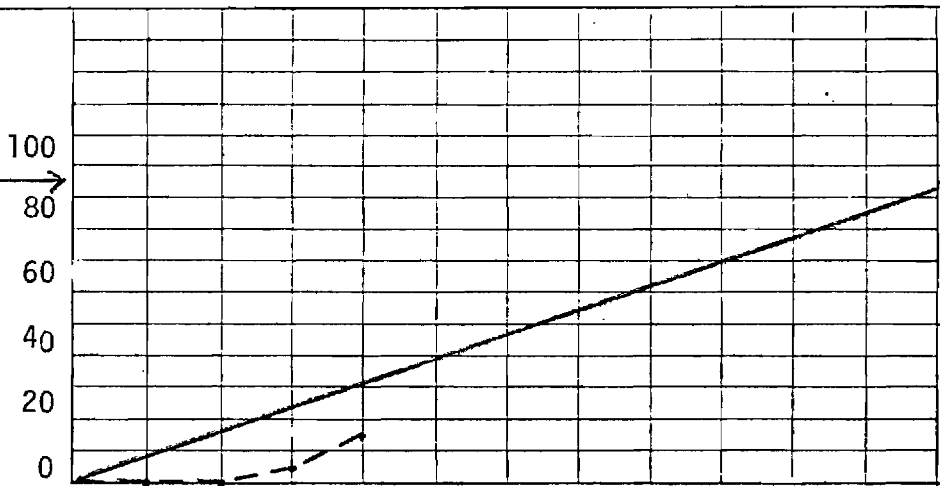
7. Months	M	J	J	A	S	O	N	D	J	F	M	A	8. FY 1978

9. Cost Status

a.

Cumulative Costs, $1,000

Total Period $83,560



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

g. Total Projected Accrued Costs for Contract: $165,613

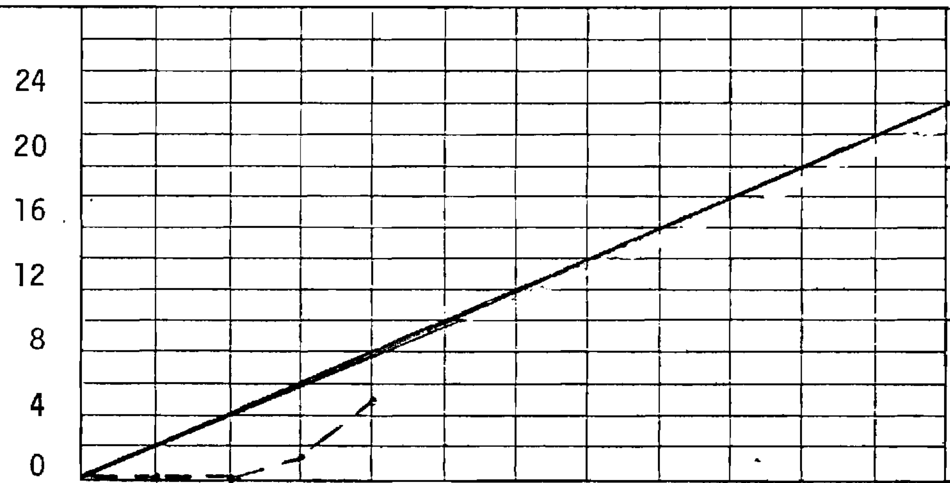
h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	6.9	13.8	20.7	27.6								
c. Actual	$K	0	0	3.8	13.8								
d. Variance	$K	6.9	13.8	16.9	13.8								

10. Manpower Status

a.

Cumulative Man Months



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

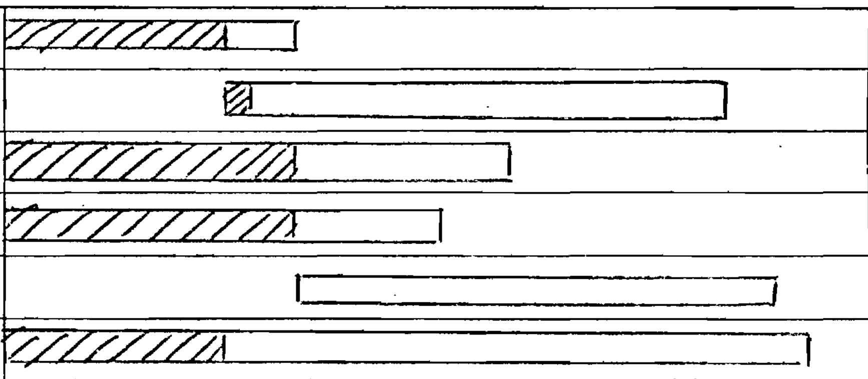
g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	2	4	6	8								
c. Actual	0	0	1.3	4.7								
d. Variance	2	4	4.7	3.3								

11. Major Milestone Status

a. Multi Region Files
b. Multi Region Analysis
c. Simplified Array Anal.
d. Array w/Storage Prog.
e. Storage Analysis
f. Experimental Studies
g.
h.



12. Remarks

13. Signature of Contractor's Project Manager and Date: 8/8/77

14. Signature of Government Technical Representative and Date

E-16-617

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 September 1977 - 30 September 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

October 8, 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - The problems with a couple of sites in the 1971 nationwide array set have been corrected and these 1971 data are now being run through the array analysis program. When this output is checked out the sorting of 1972 and 1973 data can proceed rapidly, as will the analysis of these data in multiregional and nationwide array forms.

In consultation with the contract monitor at Battelle PNL, work on the array storage analysis program is being terminated, in preparation for a substitute task on interannual variability. Preliminary examination of Local Climatological Data indicates just over 40 sites in the Continental U.S., Hawaii, Alaska and Puerto Rico which will be suitable for interannual variability study. Data coding and program development on this substitute task will begin in October.

The Grumman Windstream 25 is operational but fully automatic operation has not begun, pending further testing of a blade which seems to be producing a once per rev noise. A weld test on the suspect blade indicated no cracks in the weld seams, but further testing is planned before allowing unattended automatic operation to begin.

2. Financial and Administrative Status - Research Associate Bill Hargraves terminated with the project in August to go to graduate school at the University of Utah. Amir Mikhail was established as a post-doctoral assistant effective in September, and will be performing the duties previously assigned to Mr. Hargraves. Financial Status is as described on the attached Contract Management Summary Report.

Respectfully submitted,

C. G. Justus

FORM ERDA 536 (1/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

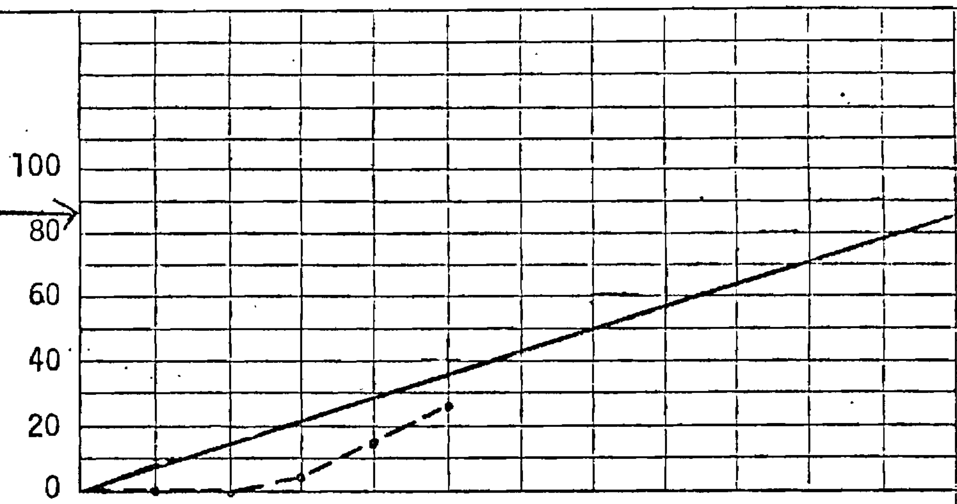
FORM APPROVED OMB NO. 38-R—

1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: ~~9/1~~ through 9/30	3. Contract Number: EY-76-5-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

7. Months: M J J A S O N D J F M A — 8. FY 1978

9. Cost Status

a. Cumulative Costs, $1,000



Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	7	14	21	28	35							
c. Actual	$K	0	0	4	14	26							
d. Variance	$K	7	14	17	14	9							

e. Actual Costs Prior FYs: $82,047

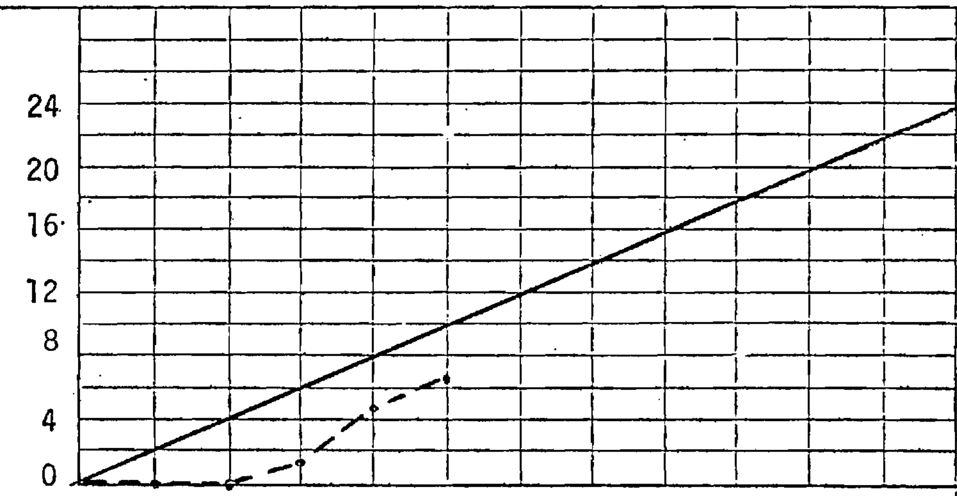
f. Planned Costs Prior FYs: $82,047

g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

10. Manpower Status

a. Cumulative Man Months



Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	2	4	6	8	10							
c. Actual	0	0	1.3	4.7	6.3							
d. Variance	2	4	4.7	3.3	3.7							

e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

11. Major Milestone Status

a. Multi Region Files

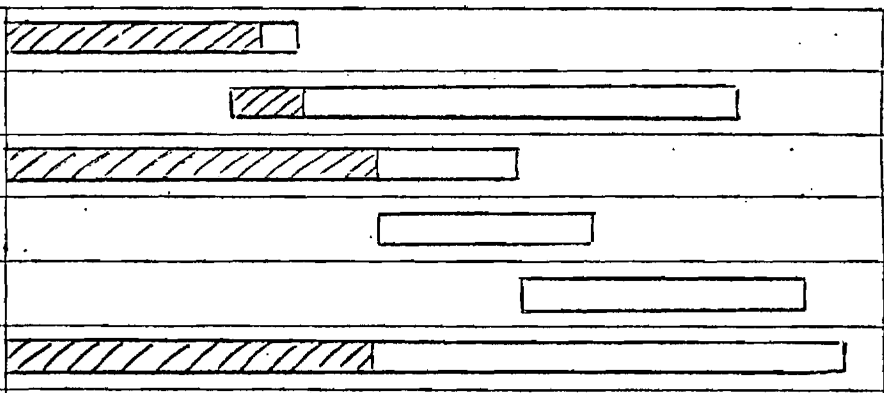
b. Multi Region Analysis

c. Simplified Array Anal.

d. Data for Interann. Var

e. Interannual Var. Anal.

f. Experimental Studies



12. Remarks

13. Signature of Contractor's Project Manager and Date: 10/3/77

14. Signature of Government Technical Representative and Date

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 October 1977 - 1 October 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

November 8, 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - Array wind data for 1972 and 1973 have been sorted and the complete nationwide array analysis runs have been done for 1971 - 1973. Runs of Northeast-plus-Great Lakes, Great Lakes-plus-Central U.S., and Central U.S.-plus-Pacific Coast are now being run. These runs, complete with cumulative 1971 - 1973 averages will complete the computer runs and only analysis, graphing, and reporting these results remains. This puts the array analysis task back on the originally proposed schedule.

   Data collection for the substitute task on interannual wind variability is well underway. Coded data from several sites is being keypunched while remaining data is being coded. The analysis program to compute probability distributions and correlations of monthly and interannual correlations has been written.

   Problems with the upper anemometer and the automatic data logger are delaying serious data acquisition with the Grumman wind turbine. Anemometer repair and replacement and data logger factory repair and replacement will take about a month. Highest power output achieved so far is about 4 kW, so there may be problems with the Grumman unit similar to those encountered by the University of Iowa. Transmission oil will be changed next month and further tests on the once per rev blade noise (which did not show to be a weld problem in those tests) will also be further investigated.

2. Financial and Administrative Status - as summarized on attached contract management summary report.

Respectfully submitted,

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

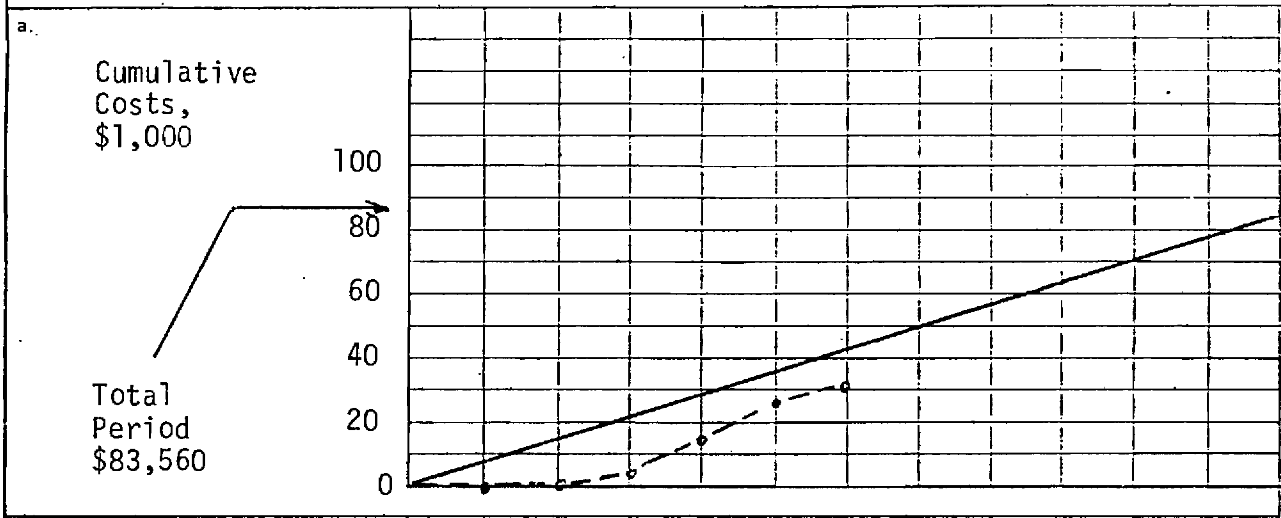
FORM APPROVED OMB NO.

1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: 10/1 through 10/31	3. Contract Number: EY-76-5-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

7. Months	M	J	J	A	S	O	N	D	J	F	M	A	8. FY 1978

9. Cost Status

a.



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

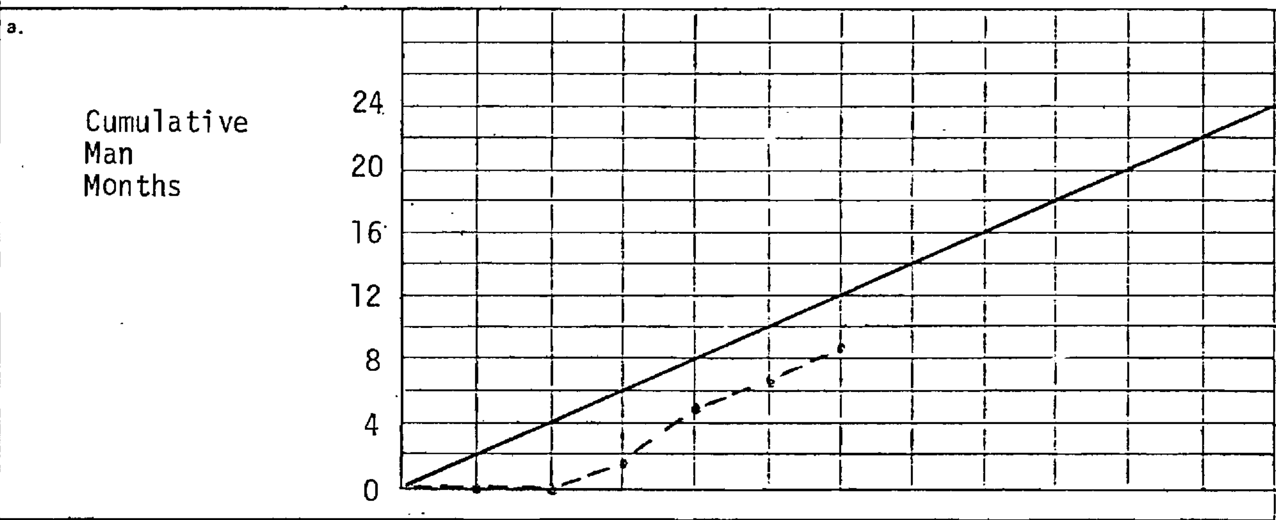
g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
	b. Planned $K	7	14	21	28	35	42						
	c. Actual $K	0	0	4	14	26	31						
	d. Variance $?	7	14	17	14	9	11						

10. Manpower Status

a.



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower		M	J	J	A	S	O	N	D	J	F	M	A
	b. Planned	2	4	6	8	10	12						
	c. Actual	0	0	1.3	4.7	6.3	8.5						
	d. Variance	2	4	4.7	3.3	3.7	3.5						

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

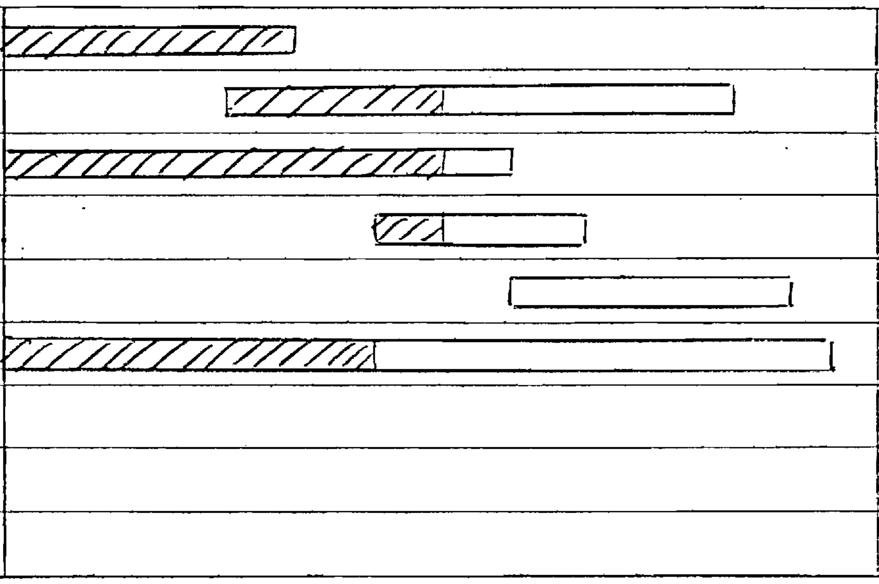
d. Data for Interann. Var

e. Interannual Var. Anal.

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date

11/8/77

14. Signature of Government Technical Representative and Date

E-16-617

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 December 1977 - 31 December 1977

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

January 8, 1978

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. Technical Status - Results for the continental arrays averaged over 1971-1973, as well as sub-arrays for Eastern plus Great Lakes, Great Lakes plus Central, and Central plus Pacific Coast have all been analyzed. Graphing and tabulating for the draft final report continues.

Preliminary runs for the 40 sites of interannual wind variability data have been done. These runs indicated the need for an averaging routine to average over results at all sites. This routine has been added to the program and results are ready to be re-run.

The rotor hub of the Grumman Windstream 25 has been sent to Grumman Houston for replacement of the pins which were provided to Grumman faulty by the pin supplier. These repairs and attendant costs of lowering and re-erecting the nacelle are being borne by Grumman. Repair and return of the rotor hub is expected in January. Re-erection of the unit and renewal of operation should begin in February. However these problems leave this experimental phase of the program somewhat behind schedule.

2. Financial and Administrative Status - as summarized on attached contract management summary report.

Respectfully submitted,

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

FORM APPROVED OMB NO.

1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays

2. Reporting Period: 12/1 through 12/31

3. Contract Number: EY-76-S-06-2439

4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332

5. Contract Start Date: 5/1/77

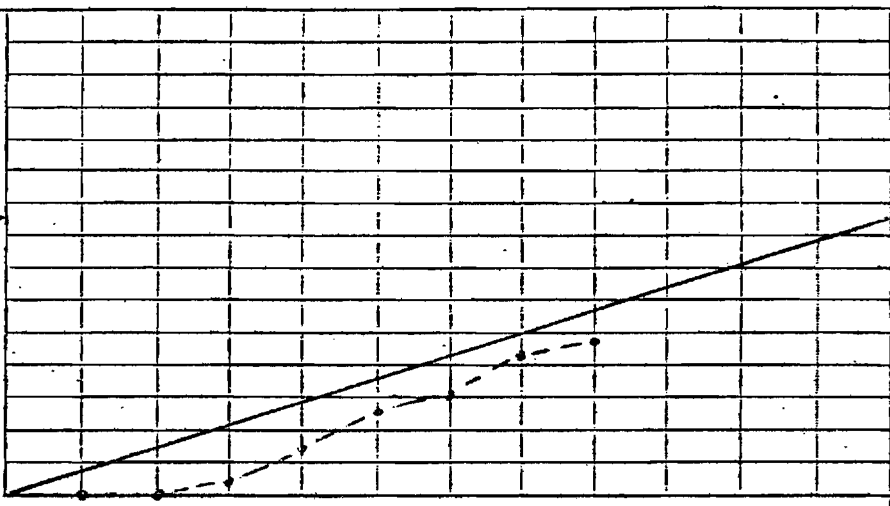
6. Contract Completion Date: 4/30/78

7. Months: M J J A S O N D J F M A

8. FY 1978

9. Cost Status

a.



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

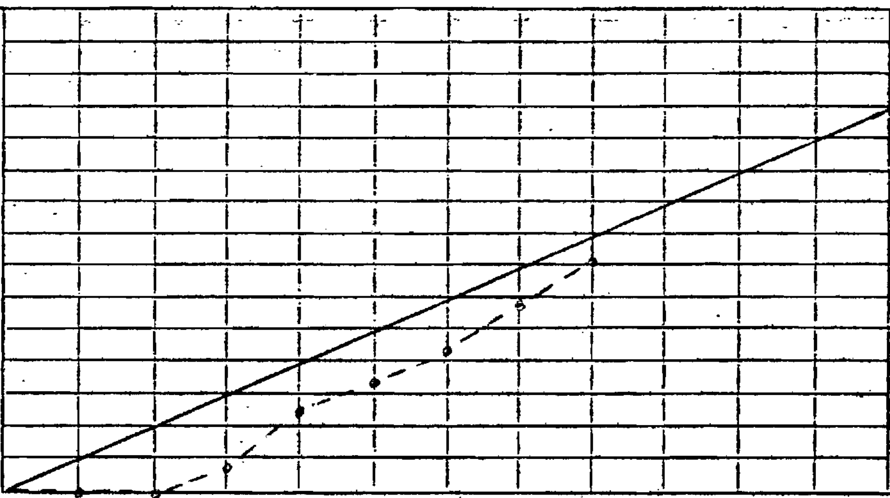
g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
	b. Planned $K	7	14	21	28	35	42	49	56				
	c. Actual $K	0	0	4	14	26	31	43	48				
	d. Variance $?	7	14	17	14	9	11	6	8				

10. Manpower Status

a.



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower		M	J	J	A	S	O	N	D	J	F	M	A
	b. Planned	2	4	6	8	10	12	14	16				
	c. Actual	0	0	1.3	4.7	6.3	8.5	11.8	14.1				
	d. Variance	2	4	4.7	3.3	3.7	3.5	2.2	1.9				

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

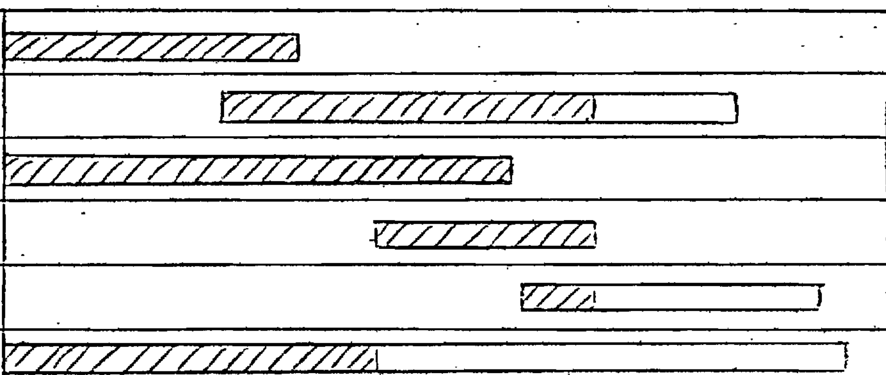
d. Data for Interann. Var.

e. Interannual Var. Anal.

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date: 1/11/78

14. Signature of Government Technical Representative and Date

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 February 1978 - 28 February 1978

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

March 8, 1978

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. <u>Technical Status</u> - Multi-Regional array analysis results are completed. An additional run will be done of the continental array to further clarify the nature of the large separation cross correlation values. As further verification of the simplified array analysis model, a run is planned using a single representative site, based on its mean wind speed. The inter-annual variations of monthly annual wind speeds have been analyzed in terms of probability distributions and correlation coefficients. An additional examination of cross correlation between sites of monthly and annual speeds will also be done. The rotor hub of the Windstream 25 was found to have additional mechanical problems after it was returned from Grumman. Grumman has decided to replace our entire unit (narcelle, blades, hub) with a new unit from their later production run. Delivery of the new unit is not anticipated until about 1 May, however, and no additional experimental data can be taken with the present unit.

2. <u>Financial and Administrative Status</u> -- as summarized on attached contract management summary report.

Respectfully submitted,

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

FORM APPROVED OMB NO.

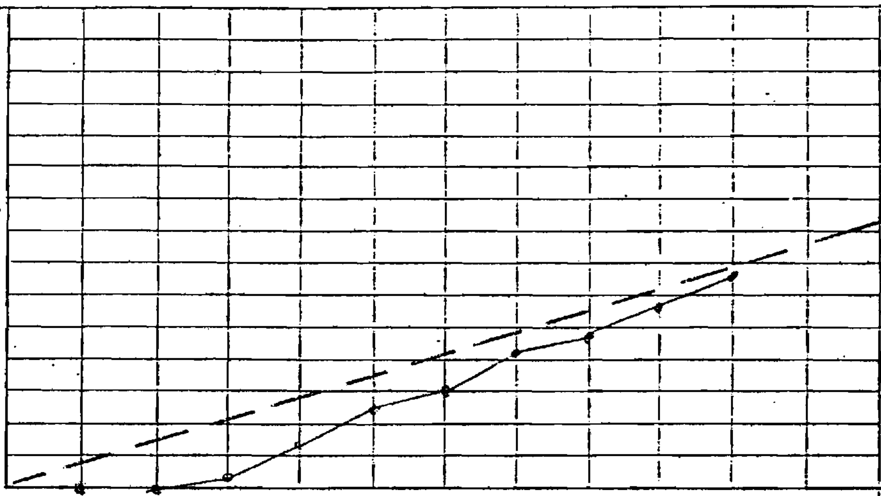
1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: 2/1 through 2/28	3. Contract Number: EY-76-S-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

7. Months: M J J A S O N D J F M A — 8. FY 1978

9. Cost Status

a. Cumulative Costs, $1,000

Total Period $83,560



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

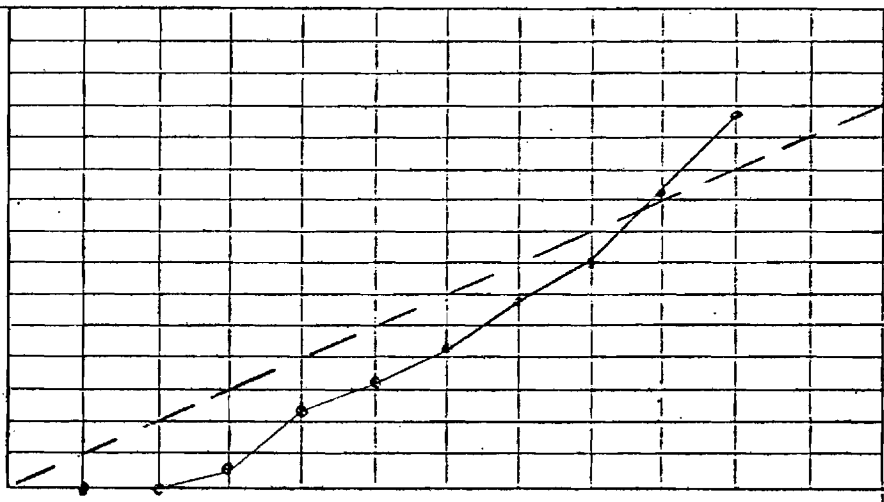
g. Total Projected Accrued Costs for Contract: $165,613

h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	7	14	21	28	35	42	49	56	63	70		
c. Actual	$K	0	0	4	14	26	31	43	48	57	68		
d. Variance	$7	7	14	17	14	9	11	6	8	6	2		

10. Manpower Status

a. Cumulative Man Months



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	2	4	6	8	10	12	14	16	18	20		
c. Actual	0	0	1.3	4.7	6.3	8.5	11.8	14.1	18.6	23.8		
d. Variance	2	4	4.7	3.3	3.7	3.5	2.2	1.9	(0.6)	(3.8)		

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

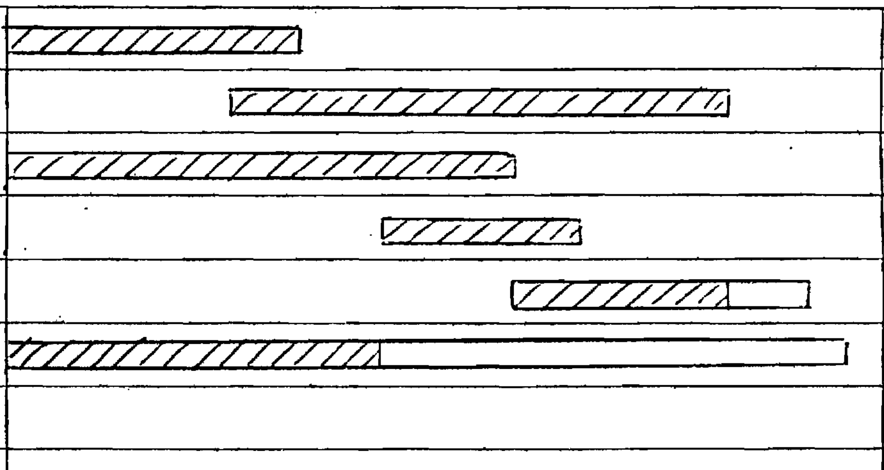
d. Data for Interann. Var.

e. Interannual Var. Anal.

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date: 3/17/78

14. Signature of Government Technical Representative and Date

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Monthly Administrative Report for the

Period: 1 March 1978 - 31 March 1978

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

April 8, 1978

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

Division of Solar Energy

UNDER CONTRACT EY-76-S-06-2439

Georgia Tech Project E-16-617

1. <u>Technical Status</u> - All analysis of the multi-regional arrays and interannual variations are complete and two separate reports on these results are being drafted and typed. The array studies report will emphasize synthesis of results from study of the several arrays which have been examined and example applications of the simplified array model relations.

2. <u>Financial and Administrative Status</u> - as summarized on attached contract management summary report.

Respectfully submitted,

C. G. Justus

FORM ERDA 536
(11/30/76)

U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

CONTRACT MANAGEMENT SUMMARY REPORT

FORM APPROVED OMB NO.

1. Contract Identification: Energy Statistics for Large Wind Turbine Arrays	2. Reporting Period: 3/1 through 3/31	3. Contract Number: EY-76-S-06-2439
4. Contractor (name and address): School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332		5. Contract Start Date: 5/1/77
		6. Contract Completion Date: 4/30/78

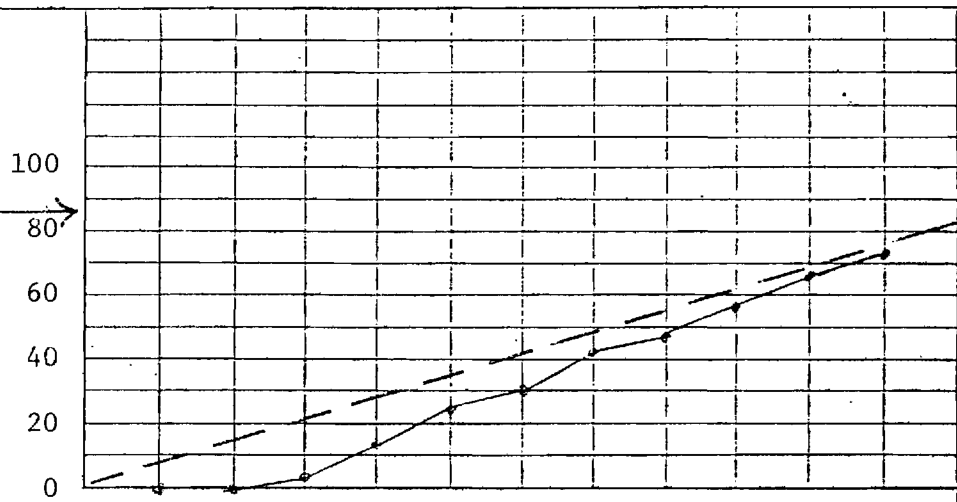
7. Months: M J J A S O N D J F M A — 8. FY 1978

9. Cost Status

a.

Cumulative Costs, $1,000

Total Period $83,560



e. Actual Costs Prior FYs: $82,047

f. Planned Costs Prior FYs: $82,047

g. Total Projected Accrued Costs for Contract: $165,613

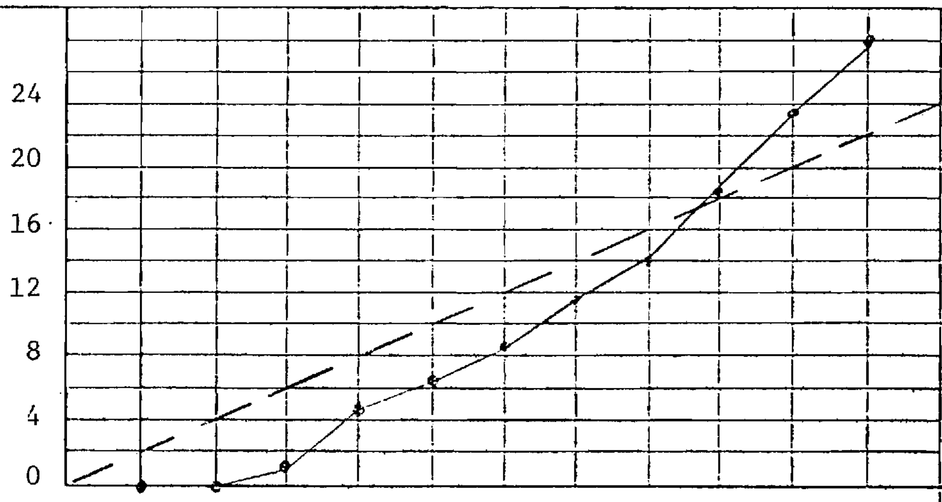
h. Total Contract Value: $165,613

Accrued Costs		M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	$K	7	14	21	28	35	42	49	56	63	70	77	
c. Actual	$K	0	0	4	14	26	31	43	48	57	68	73	
d. Variance	$K	7	14	17	14	9	11	6	8	6	2	4	

10. Manpower Status

a.

Cumulative Man Months



e. Actual Manpower Prior FYs: 24.8 man months

f. Planned Manpower Prior FYs: 22 man months

g. Total Projected Manpower for Contract: 48.8 man months

h. Total Contract Manpower: 46 man months

Manpower	M	J	J	A	S	O	N	D	J	F	M	A
b. Planned	2	4	6	8	10	12	14	16	18	20	22	
c. Actual	0	0	1.3	4.7	6.3	8.5	11.8	14.1	18.6	23.8	28.8	
d. Variance	2	4	4.7	3.3	3.7	3.5	2.2	1.9	(0.6)	(3.8)	(6.8)	

11. Major Milestone Status

a. Multi Region Files

b. Multi Region Analysis

c. Simplified Array Anal.

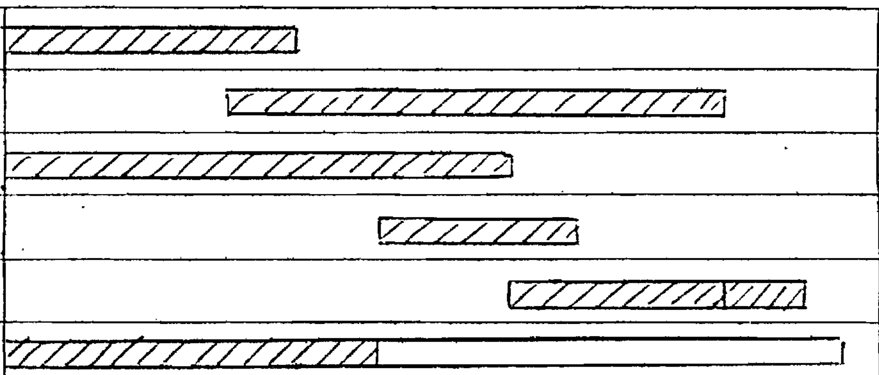
d. Data for Interann. Var.

e. Interannual Var. Anal.

f. Experimental Studies

g.

h.



12. Remarks

13. Signature of Contractor's Project Manager and Date: 3/17/78

14. Signature of Government Technical Representative and Date

RLO/2439-77/2

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ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Quarterly Progress Report

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta GA 30332

Date Published - August 1977

Report Period 1 May 1977 - 31 July 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

DIVISION OF SOLAR ENERGY

UNDER CONTRACT EY-76-5-06-2439

Georgia Tech Project E-16-617

PROJECT OBJECTIVES

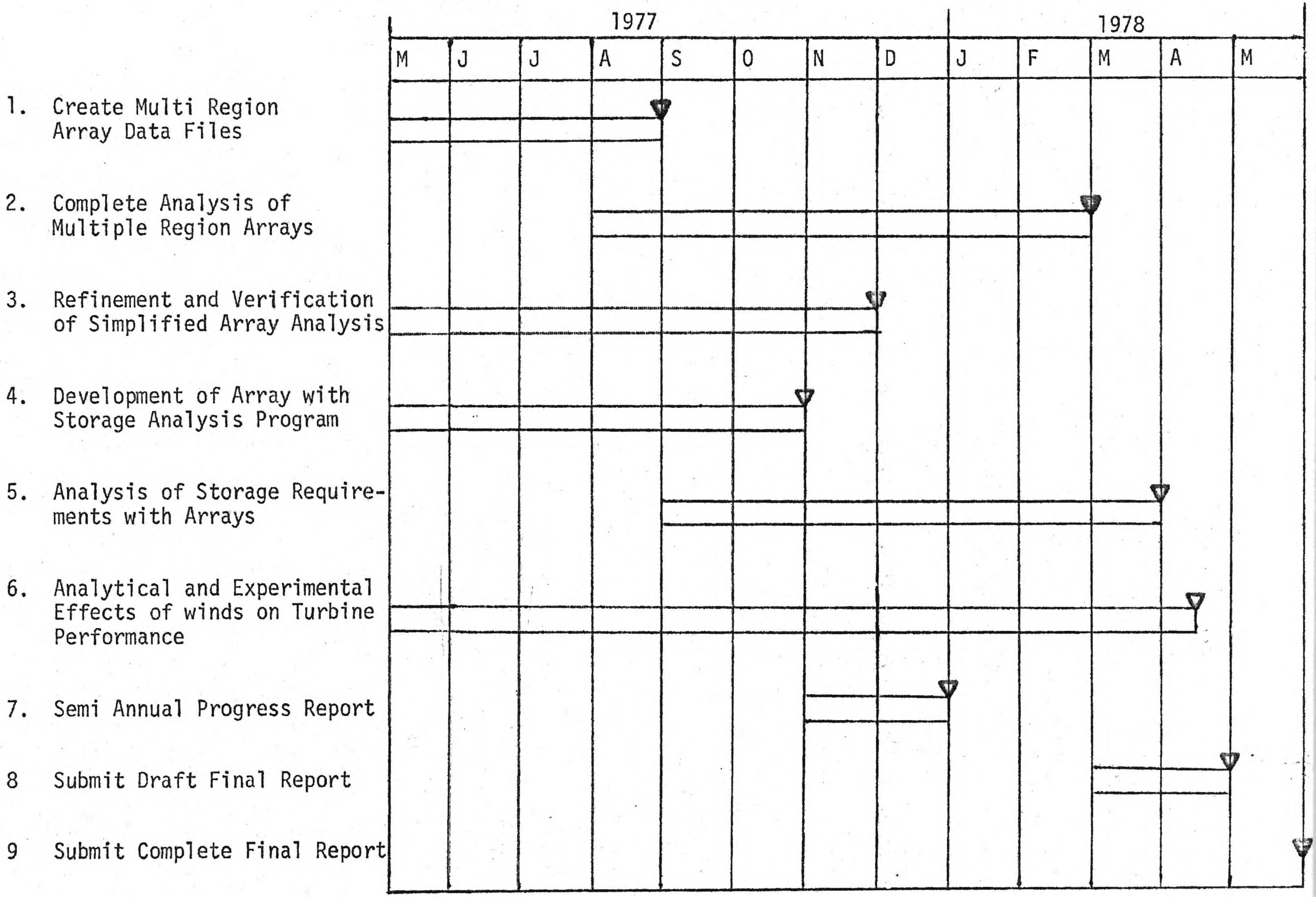
The original studies on wind energy statistics for large arrays of wind turbines examined the New England and Central U.S. areas. In the current funding period additional array analysis is being done in the Great Lakes area and Pacific Coast regions. These array analyses produce information on time autocorrelation, spatial cross correlation, mean, seasonal and diurnal wind power output, wind power frequency distribution (reliability without storage), and wind power return times (duration of runs). New areas of investigation presently underway include simulation of the effects of wind gusts and wind shear on the power performance curve used to simulate wind turbine power output, correlation of degree days (heating or cooling) with diurnal average wind power, and installation of a wind turbine for use in experimental testing of these meteorological effects on power performance curves. Statistical methods are also being developed whereby the wind power output of large arrays can be simulated from simple site statistics without resort to costly and time consuming time series analysis. Array analysis will continue with emphasis on: larger arrays (multiregion) to take maximum benefit of diversity, refinement and verification of the simplified array simulation not requiring time series analysis of many sites, time series array analysis of storage with explicit simulation of storage charge and discharge cycling.

TECHNICAL PLAN

The technical effort of this project is divided into four tasks as outlined below. The project schedule and milestone chart is shown in Figure 1.

1. (a) Evaluate array wind performance statistics of large multiregional arrays having maximum wind diversity. This will be done using selected sites from among multiple regions previously

Figure 1 - Milestone Chart



studied individually.

(b) Continue development, verification, and refinement of simplified methods for simulating large array performance from single example representative site statistics, in order to provide engineering purpose array analysis methods which do not require extensive and costly analysis of time series data from many simultaneous sites.

2. Study the enhanced power availability from large array with storage. Wind diversity over the array will mean that less storage will be required to achieve a required reliability than would be inferred from analysis of a single site with storage. The magnitude of these effects will be computed from time series storage analysis methods, keeping track of storage charge and discharge cycles (i.e. more complete than just the run duration statistics previously evaluated).

3. In order to improve the modeling of wind turbine power output, simple analytical models of effects of wind shear, wind gust and direction shifts on power output will be investigated.

4. Parallel with Task 3, experimental studies will be conducted with the Grumman Windstream 25, currently being installed on the Georgia Tech campus with joint Tech-ERDA funds.

PROGRESS TO DATE

Tape data on hand plus additional data acquired from the National Climatic Center in Asheville, have been sorted into a large nationwide array of wind data for array studies. The sites in this array are listed in Table 1. Data for the years 1971 - 1973 will be studied. Runs of a simulated 2 MW wind turbine have been done for 1971 and some of the results are shown in Figures 2 and 3. Figure 2 shows the average array cross correlation versus site separation. Figure 3 shows computed frequency of array output power and the simple array model simulated output power (refinement of which will continue as Task 1 (b) (milestone 3).

The results in Figure 2 show that annual average correlations do not go significantly negative, even at the largest separations, even though certain site pair correlations do go negative for some months in the 1971 year shown. Note the change in horizontal scale at 1000 km.

The results of Figure 3 show that power levels of 500 kW per 2 MW generator are approximately 90% reliable over the year 1971. The 1971 hourly data show on an annual basis approximately 95-99% reliability of 500 kW/generator throughout the hours 10 am - 7 pm (corresponding to peak load periods). These reliability levels are significantly enhanced by the diversity in the nationwide array, for which the average individual site power output of 500 kW would be only about 60%.

Despite the late start of project renewal initiation, which means that no work could begin prior to July, the project is now essentially back on the original proposed schedule.

Table 1

Large Scale Array Sites

Site Name	Site Code	
Atlantic City, NJ	ACY	93730
Boston, MA	BOS	14739
Laguardia, NY	LGA	14732
Providence, RI	PVD	14765
Blue Hill, MA	BLU	14753
Grand Island, NB	GRI	14935
Amarillo, TX	AMA	23047
Concordia, KS	CNK	13984
Dodge City, KS	DDC	13985
Good Land, KS	GLD	23065
Russel, KS	RSL	93997
Oklahoma Cityk OK	OKC	13967
Buffalo, NY	BUF	14733
Detroit (metro), MI	DTW	94847
Erie, PA	ERI	14860
Mansfield, OH	MFD	14891
Milwaukee, WI	MKE	14839
Muskegon, WI	MKE	14840
North Bend, OR	OTH	24284
Sandberg, CA	SDB	23187
San Francisco, CA	SFL	23234
Stampede Pass, WA	SMP	24237

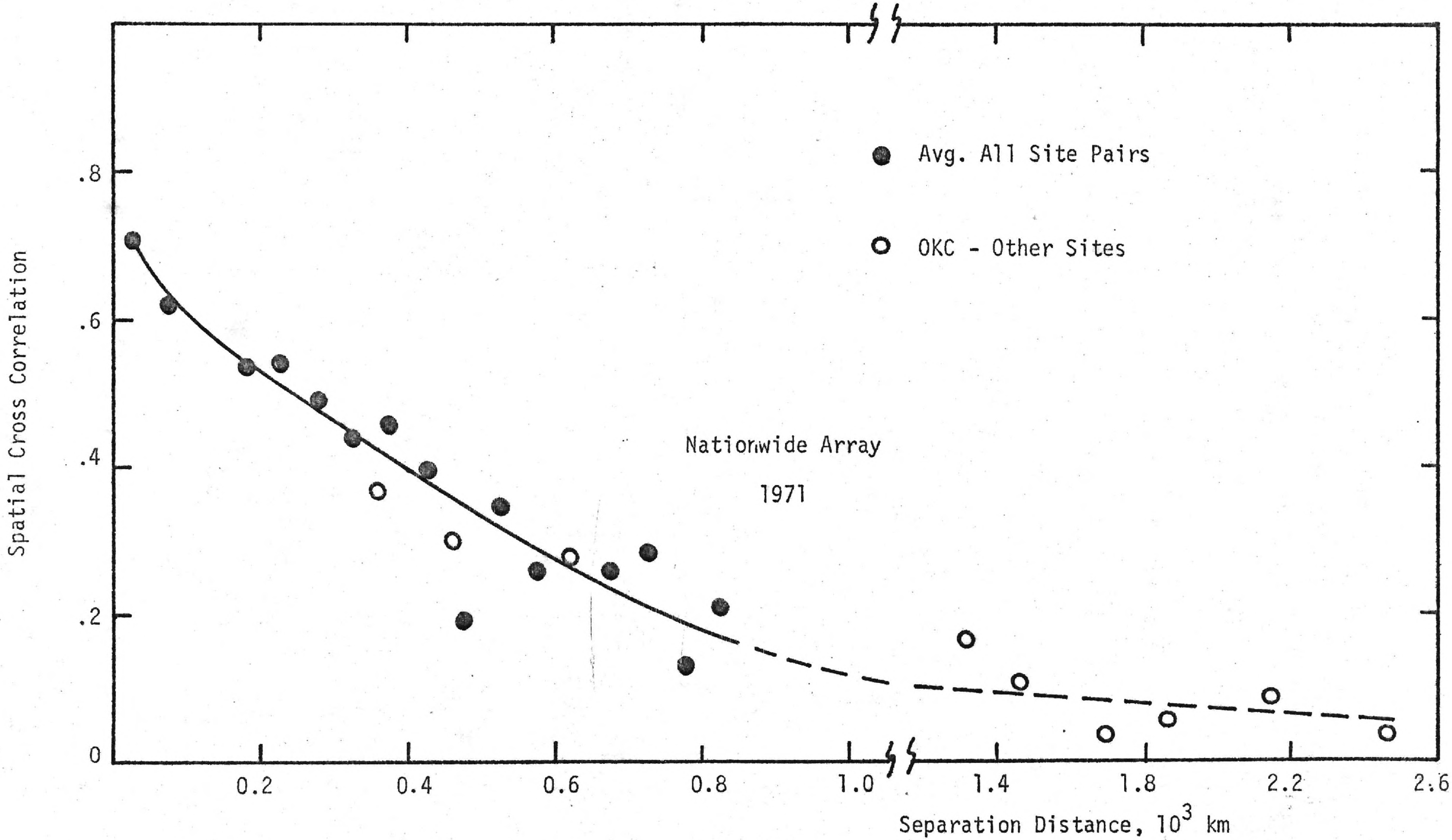


Figure 2 - Average Cross Correlation Versus Spatial Separation for the Nationwide Array (Table 1) for 1971.

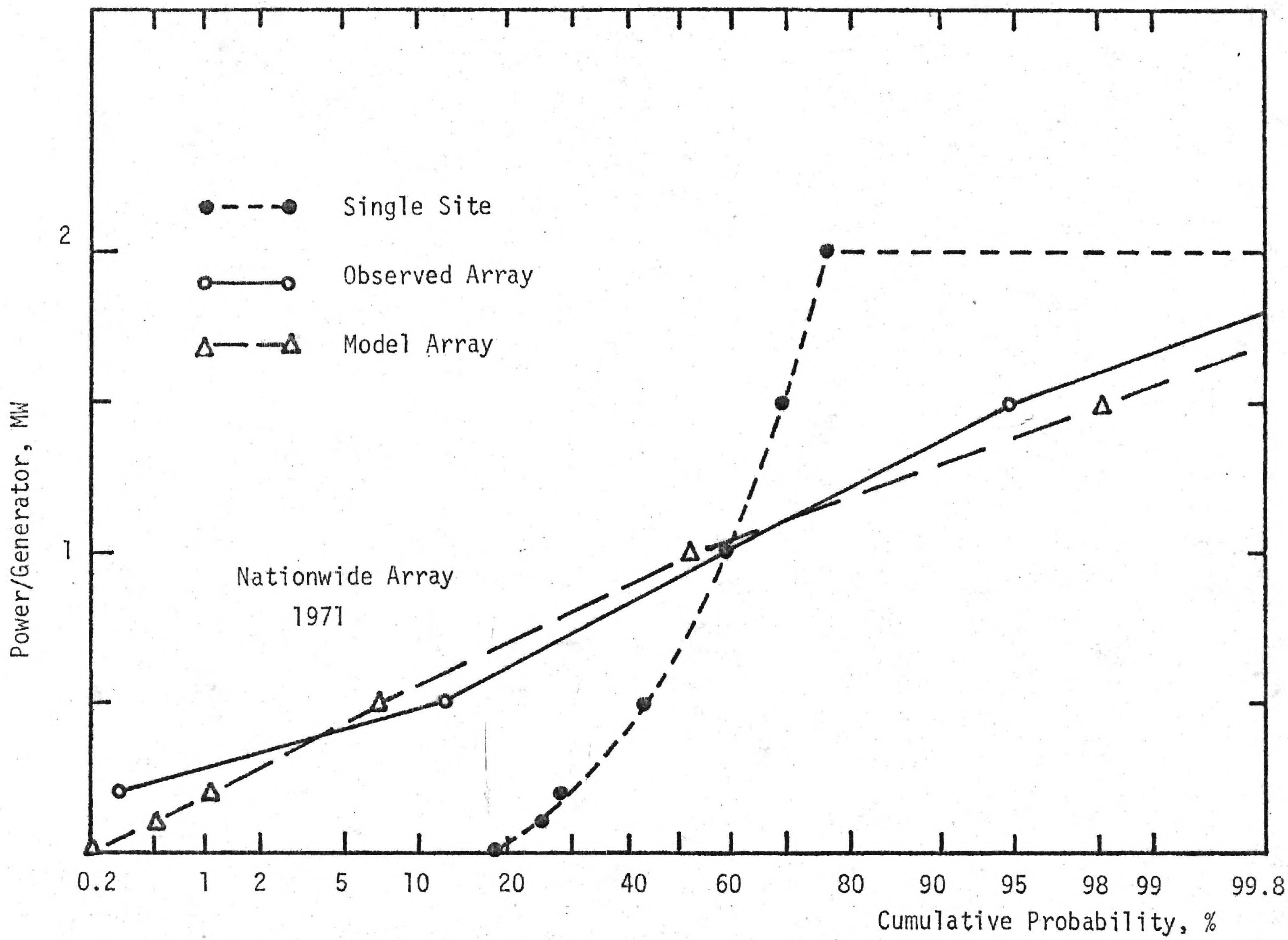


Figure 3 - Computed Average Single Site and Array Power Output Probability and Modeled Array Power Output Probability for Nationwide Array, 1971.

Form ERDA-427
(2675)
ERDAM 3201

U.S. ENERGY RESEARCH & DEVELOPMENT ADMINISTRATION

UNIVERSITY–TYPE CONTRACTOR'S RECOMMENDATION FOR DISPOSITION OF SCIENTIFIC AND TECHNICAL DOCUMENT

(See Instructions on Reverse Side)

1. ERDA REPORT NO.	2. TITLE
RLO/2439-77/3	Energy Statistics for Large Wind Turbine Arrays

3. TYPE OF DOCUMENT (Check one):

- ☐ a. Scientific and technical report
- ☐ b. Conference paper:
  - Title of conference ____
  - Date of conference ____
  - Exact location of conference ____
  - Sponsoring organization ____
- ☒ c. Other (Specify) Quarterly Progress Report

4. RECOMMENDED ANNOUNCEMENT AND DISTRIBUTION (Check one):

- ☐ a. ERDA's normal announcement and distribution procedures may be followed.
- ☒ b. Make available only within ERDA and to ERDA contractors and other U.S. Government agencies and their contractors.

5. REASON FOR RECOMMENDED RESTRICTIONS:

Report contains only preliminary results

6. SUBMITTED BY: NAME AND POSITION (Please print or type)

C. G. Justus, Professor

Organization

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Signature	Date
	December 7, 1977

FOR ERDA USE ONLY

7. ERDA CONTRACT ADMINISTRATOR'S COMMENTS, IF ANY, ON ABOVE ANNOUNCEMENT AND DISTRIBUTION RECOMMENDATION:

8. PATENT CLEARANCE:

- ☐ a. ERDA patent clearance has been granted by responsible ERDA patent group.
- ☐ b. Report has been sent to responsible ERDA patent group for clearance
- ☐ c. Patent clearance not required.

RLO/2439-77/3

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Quarterly Progress Report

C. G. Justus

School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Date Published - December 1977

Report Period 1 August 1977 - 31 November 1977

PREPARED FOR THE UNITED STATES
ENERGY RESEARCH AND DEVELOPMENT
ADMINISTRATION

DIVISION OF SOLAR ENERGY

UNDER CONTRACT EY-76-5-06-2439

Georgia Tech Project E-16-617

PROJECT OBJECTIVES

Previous studies on wind energy statistics for large arrays of wind turbines examined arrays in New England, Central U.S., Great Lakes, and Pacific Coast areas. These array analyses produce information on time autocorrelation, spatial cross correlation, mean, seasonal and diurnal wind power output, wind power frequency distribution (reliability without storage), and wind power run durations (length of power outage periods). Statistical methods have been developed whereby the wind power output of large arrays can be simulated from simple site statistics without resort to costly and time consuming time series analysis. Array analysis is continuing with emphasis on: larger arrays (multiregion) to take maximum benefit of diversity, refinement and verification of the simplified array simulation not requiring time series analysis of many sites. Parallel experimental studies are being conducted with a Grumman Windstream 25 installed on the Georgia Tech campus. A study of inter-annual variability of monthly mean winds has been substituted for the originally proposed study of arrays with storage.

TECHNICAL PLAN

The technical effort of this project is divided into four tasks as outlined below. The project schedule and milestone chart is shown in Figure 1.

1. (a) Evaluate array wind performance statistics of large multi-regional arrays having maximum wind diversity. This will be done using selected sites from among multiple regions previously studied individually.

   (b) Continue development, verification, and refinement of simplified methods for simulating large array performance from single example representative site statistics, in order to provide engineering purpose array analysis methods which do not require extensive and costly analysis of time series data from many simultaneous sites.

2. The inter-annual variation of monthly mean wind speed will be studied from about 40 U.S. sites which have 10-12 or more years of data from a constant anemometer location. Probability distributions and time autocorrelations will be examined.

3. In order to improve the modeling of wind turbine power output, simple analytical models of effects of wind shear, wind gust and direction shifts on power output will be investigated.

4. Parallel with Task 3, experimental studies will be conducted with the Grumman Windstream 25, currently being installed on the Georgia Tech campus with joint Tech-ERDA funds.

The present Task 2 has been substituted for the previously proposed study of array power output statistics for arrays with storage.

PROGRESS TO DATE

The full continental array plus sub-arrays for Eastern plus Great Lakes, Great Lakes plus Central, and Central plus Pacific Coast have been analyzed and averaged for the years 1971 through 1973. These results are being tabulated, graphed and examined in preparation for their reporting. Sample results, for the full continental array, are given in Tables 1 - 7.

Data collection of monthly mean winds from approximately 40 sites with mean winds $\geq$ 5 m/s (11 mph) has been essentially completed. The program has been written and checked out which will be used to compute probability distribution of interannual changes in monthly mean winds, and time autocorrelation of monthly mean winds. Computer computations will begin next month.

Experimental studies with the Grumman Windstream 25 have been hampered by electronic problems in the primary data logger and tape deck interface. Some data have been obtained on the back-up strip chart recorders and analysis of these results will begin soon. A further set back is the re-call by Grumman of the rotor hub assembly to replace blade retaining pins which have been found faulty (not provided up to MIL-SPEC'S by the pin supplier). The nacelle will be lowered from the tower soon, the blades removed and the rotor hub assembly shipped to Grumman for replacement of these faulty pins.

Figure 1 - Milestone Chart

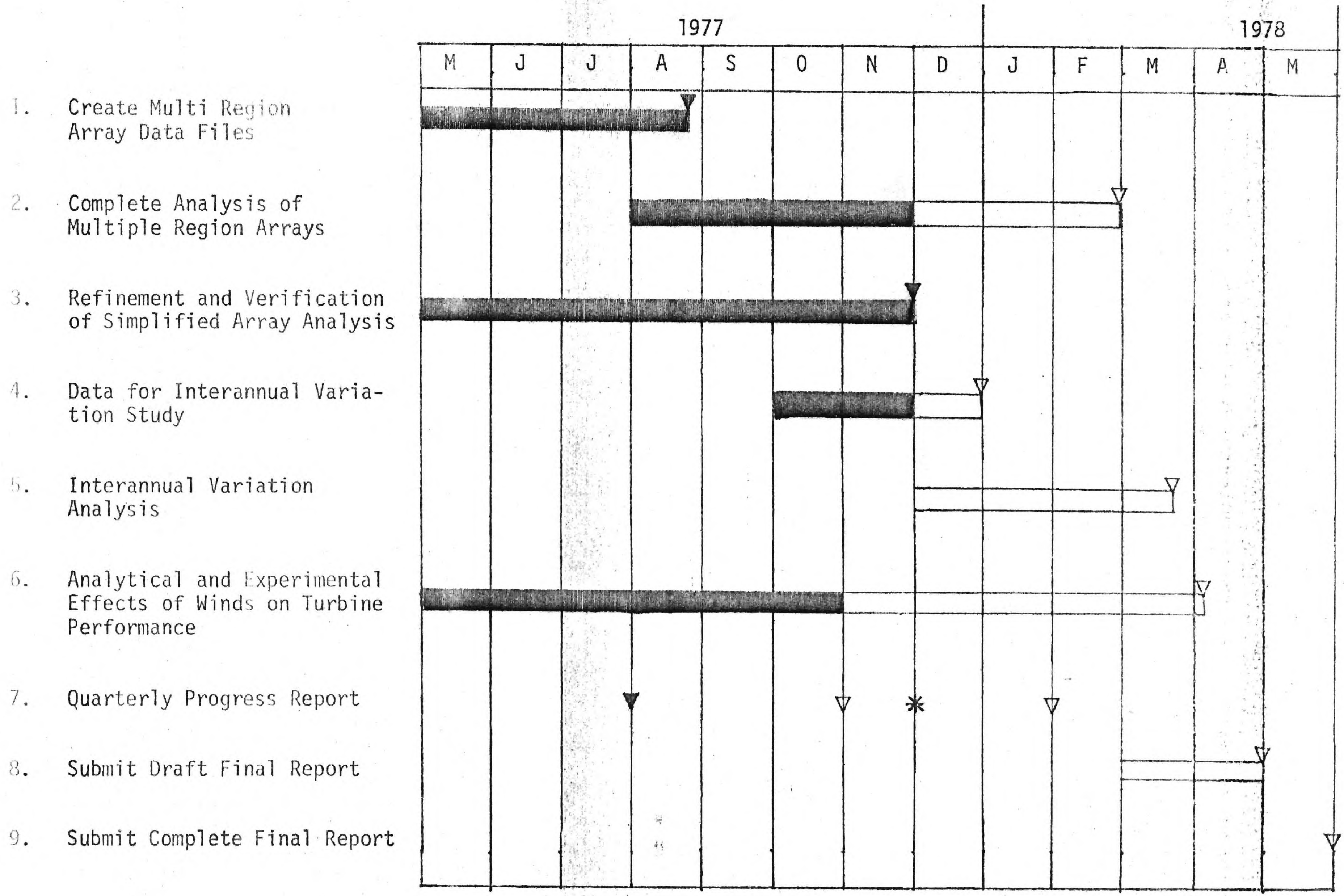


Table 1

Continental Array Sites

NORTHEAST

ACY	Atlantic City, NJ
BOS	Boston, MA
BLU	Bluehill, MA
LGA	New York (Laguardia), NY
PVD	Providence, RI

GREAT LAKES

BUF	Buffalo, NY
DTW	Detroit, MI
ERI	Erie, PA
MFD	Mansfield, OH
MKE	Milwaukee, WI
MKG	Muskegon, MI

CENTRAL U.S.

AMA	Amarillo, TX
CNK	Concordia, KS
DDC	Dodge City, KS
GLD	Goodland, KS
GRI	Grand Island, KS
OKC	Oklahoma City, OK
RSL	Russell, KS

PACIFIC COAST

OTH	North Bend, OR
SDB	Sandberg, CA
SFO	San Francisco, CA
SMP	Stampede Pass, WA

Table 2

Mean Wind Speed (m/s) at 60 m (197 ft) Hub Height for Continental Array Sites

	Winter	Spring	Summer	Fall	Annual
ACY	7.9	8.3	6.5	6.7	7.3
AMA	9.1	10.4	8.9	9.1	9.4
BLU	12.4	11.5	9.7	10.6	11.1
BOS	9.1	8.0	6.5	7.9	7.9
BUF	9.4	8.2	7.2	7.4	8.1
CNK	8.5	9.1	7.9	8.1	8.4
DDC	8.8	9.7	8.3	8.4	8.8
DTW	8.3	7.9	6.4	7.0	7.4
ERI	9.1	8.1	6.8	7.6	7.9
GLD	8.9	9.8	8.6	8.7	9.0
GRI	8.2	9.1	7.4	7.8	8.2
LGA	9.1	8.9	7.3	7.9	8.3
MFD	9.1	8.5	6.6	7.4	7.9
MKE	9.0	8.5	7.5	7.9	8.2
MKG	8.9	8.0	6.5	7.5	7.7
OKC	8.8	9.5	7.5	8.0	8.5
OTH	7.0	7.5	7.7	6.7	7.2
PVD	8.0	8.0	6.6	6.8	7.3
RSL	8.3	9.0	8.3	7.8	8.4
SDB	8.9	9.2	7.3	7.9	8.2
SFO	5.7	8.0	8.4	6.7	7.2
SMP	8.1	7.9	7.6	7.5	7.8
	---	---	---	---	---
AVG.	8.7	8.8	7.5	7.8	8.2

Table 3

Seasonal and Diurnal Variations of 60 m (197 ft) Hub Height Mean Wind Speed (m/s) for Continental Array

Season / Hour	1	4	7	10	13	16	19	22	Avg.
Winter	8.4	8.3	8.2	8.7	9.4	9.3	8.5	8.4	8.7
Spring	8.0	7.8	7.9	9.1	9.8	10.1	9.3	8.2	8.8
Summer	6.9	6.5	6.5	7.6	8.5	9.0	8.3	7.1	7.5
Fall	7.3	7.1	7.1	8.1	8.7	8.8	7.8	7.5	7.8
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Annual	7.6	7.4	7.5	8.4	9.1	9.3	8.4	7.8	8.2

Table 4

Seasonal and Diurnal Variations of Mean Output Power from Continental Array of 1.5 MW Wind Turbines

Season \ Hour	1	4	7	10	13	16	19	22	Avg.
Winter	469	457	451	519	614	604	474	474	508
Spring	400	386	393	566	651	691	564	418	509
Summer	225	192	181	310	408	486	390	342	304
Fall	300	281	286	397	479	488	351	317	362
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Annual	348	329	328	448	538	567	445	363	421

Table 5

Seasonal and Diurnal Variations of Mean Output Power from Continental Array of 2.0 MW Wind Turbines

Seasonal / Hour	1	4	7	10	13	16	19	22	Avg.
Winter	977	961	953	1063	1196	1186	982	979	1037
Spring	877	848	875	1128	1266	1347	1159	909	1051
Summer	601	534	529	783	984	1100	910	628	759
Fall	722	686	694	904	1034	1062	816	753	834
	---	---	---	---	----	----	---	---	---
Annual	794	757	763	970	1120	1174	967	817	920

Table 6

Availability (percent) of 200 kW per 2 MW Generator in Continental Array

(Individual Site and Whole Array)

Hour	Winter		Spring		Summer		Fall		Annual	
	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	73.9	100.0	71.1	99.6	58.2	99.6	63.7	99.6	66.7	99.7
4	73.3	100.0	68.9	100.0	53.9	98.9	62.0	99.3	64.5	99.5
7	73.0	100.0	70.9	100.0	55.3	97.1	62.7	99.3	63.4	99.1
10	78.0	100.0	80.0	100.0	68.9	99.6	73.3	100.0	75.0	99.1
13	82.4	100.0	85.8	100.0	79.8	100.0	78.2	100.0	81.6	100.0
16	82.2	99.6	89.1	100.0	84.2	100.0	81.1	100.0	84.2	99.9
19	74.2	100.0	83.4	100.0	76.1	100.0	68.5	100.0	75.6	100.0
22	73.9	100.0	71.6	99.6	58.5	100.0	65.8	100.0	67.4	99.9
All	76.4	100.0	77.6	99.9	66.9	99.4	69.4	99.8	72.5	99.8

Table 7

As in Table 6 for 500 kW per 2 MW Generator in Continental Array

Hour	Winter		Spring		Summer		Fall		Annual	
	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	60.0	95.2	54.4	89.5	40.4	68.8	45.9	82.1	50.1	83.9
4	58.9	94.5	52.6	87.3	37.0	52.9	44.3	72.9	48.1	76.8
7	59.0	94.8	55.4	88.4	37.1	52.9	45.1	74.7	49.0	77.6
10	64.7	97.4	68.4	98.2	53.2	88.0	58.3	91.2	61.1	93.7
13	70.9	98.9	75.0	100.0	66.2	99.3	65.0	98.2	69.3	99.1
16	70.6	98.2	80.4	100.0	71.9	99.3	66.6	91.4	72.6	98.7
19	59.4	97.0	70.2	99.6	60.0	99.3	51.9	90.5	60.4	96.6
22	58.9	97.8	56.2	91.3	42.3	73.2	47.7	82.1	51.2	86.0
All	62.8	96.7	64.1	94.3	51.0	79.2	53.1	86.1	57.7	89.1

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Quarterly Progress Report

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PROJECT OBJECTIVES

Previous studies on wind energy statistics for large arrays of wind turbines examined arrays in New England, Central U.S., Great Lakes, and Pacific Coast areas. These array analyses produce information on time autocorrelation, spatial cross correlation, mean, seasonal and diurnal wind power output, wind power frequency distribution (reliability without storage), and wind power run durations (length of power outage periods). Statistical methods have been developed whereby the wind power output of large arrays can be simulated from simple site statistics without resort to costly and time consuming time series analysis. Array analysis is continuing with emphasis on: larger arrays (multiregion) to take maximum benefit of diversity, refinement and verification of the simplified array simulation not requiring time series analysis of many sites. Parallel experimental studies are being conducted with a Grumman Windstream 25 installed on the Georgia Tech campus. A study of inter-annual variability of monthly mean winds has been substituted for the originally proposed study of arrays with storage.

TECHNICAL PLAN

The technical effort of this project is divided into four tasks as outlined below. The project schedule and milestone chart is shown in Figure 1.

1. (a) Evaluate array wind performance statistics of large multi-regional arrays having maximum wind diversity. This will be done using selected sites from among multiple regions previously studied individually.

   (b) Continue development, verification, and refinement of simplified methods for simulating large array performance from single example representative site statistics, in order to provide engineering purpose array analysis methods which do not require extensive and costly analysis of time series data from many simultaneous sites.

2. The inter-annual variation of monthly mean wind speed will be studied from about 40 U.S. sites which have 10-12 or more years of data from a constant anemometer location. Probability distributions and time autocorrelations will be examined.

3. In order to improve the modeling of wind turbine power output, simple analytical models of effects of wind shear, wind gust and direction shifts on power output will be investigated.

4. Parallel with Task 3, experimental studies will be conducted with the Grumman Windstream 25, currently being installed on the Georgia Tech campus with joint Tech-ERDA funds.

The present Task 2 has been substituted for the previously proposed study of array power output statistics for arrays with storage.

PROGRESS TO DATE

The full continental array plus sub-arrays for Eastern plus Great Lakes, Great Lakes plus Central, and Central plus Pacific Coast have been analyzed and averaged for the years 1971 through 1973. These results have been tabulated, graphed and examined in preparation for their reporting. Sample results, for the full continental array, were given in the previous quarterly report.

Data collection of monthly mean winds from approximately 40 sites with mean winds $\geq$ 5 m/s (11 mph) is complete. The program has been run which computes the probability distribution of interannual changes in monthly mean winds, and time autocorrelation of monthly mean winds. Tables 1-3 give site names, codes, mean winds, anemometer heights, and length of data sample for the sites used in the interannual variation study. Sample results are given in Figures 1 and 2. Figure 1 shows that, on average over all 40 sites, and expressed as ratios relative to long term mean wind $\bar{V}$, 10% of annual mean winds are less than 0.91 $\bar{V}$, while 90% of annual mean winds are less than 1.07 $\bar{V}$. For monthly mean winds, Figure 1 shows 10% to be less than 0.83 $\bar{V}$ while 90% are less than 1.12 $\bar{V}$. Figure 2 gives average time autocorrelations of monthly mean wind and annual mean wind, averaged over all 40 of the sites. This figure shows a roughly 1 in 3 chance that two consecutive months will be consistantly high or low relative to the long term mean, with similar probability of two consecutive high or low annual means.

Experimental studies with the Grumman Windstream 25 have been hampered by electronic problems in the primary data logger and tape deck interface. A further set back is the re-call by Grumman of the rotor hub assembly to replace blade retaining pins which have been found faulty (not provided up to MIL-SPEC'S by the pin supplier). The nacelle has been lowered from the tower, the blades removed and the rotor hub assembly has been repaired by Grumman by replacement of these faulty pins. Re assembly and commencement of operation with data logger recording should begin next month.

Table 1

List of Sites, by States, for Interannual Variability Study

State	Site
Alaska	Barter Island
	Bethel
	Cold Bay
	King Salmon
	St. Paul Island
California	Sandberg
Connecticut	Bridgeport
Florida	Key West
Hawaii	Honolulu
	Kahului
	Lihue
Illinois	Springfield
Iowa	Des Moines
Kansas	Concordia
	Dodge City
Massachusetts	Boston
Minnesota	Duluth
	Rochester
Missouri	Springfield
Montana	Billings
	Great Falls
Nebraska	Grand Island
New Hampshire	Mt. Washington
New York	Buffalo
	New York (JFK)
	New York (La Guardia)
North Carolina	Cape Hatteras
North Dakota	Fargo
Pacific Ocean	Wake Island
South Dakota	Aberdeen
	Huron
	Sioux Falls
Texas	Amarillo
	Corpus Christi
	Ft. Worth
	Waco
	Wichita Falls
Wisconsin	Milwaukee
Wyoming	Casper
	Cheyenne

Table 2

Site Codes for Interannual Variability Study

CODE	STATION	CODE	STATION
ABR	Aberdeen, SD	FSD	Sioux Falls, SD
ACT	Waco, TX	GRI	Grand Island, NB
AKN	King Salmon, AK	GSW	Ft. Worth, TX
AMA	Amarillo, TX	GTF	Great Falls, MT
BDR	Bridgeport, CT	HAT	Cape Hatteras, NC
BET	Bethel, AK	HLI	Lihue, HI
BIL	Billings, MT	HNL	Honolulu, HI
BOS	Boston, MA	HOG	Kahului, HI
BTI	Barter Island, AK	HON	Huron, SD
BUF	Buffalo, NY	JFK	New York (JFK), NY
CDB	Cold Bay, AK	LGA	New York (La Guardia), NY
CNK	Concordia, KA	MKE	Milwaukee, WI
CPR	Casper, WY	MWN	Mt. Washington, NH
CRP	Corpus Christi, TX	RST	Rochester, MN
CYS	Cheyenne, WY	SDB	Sandberg, CA
DDC	Dodge City, KA	SGF	Springfield, MO
DLH	Duluth, MN	SNP	St. Paul Island, AK
DSM	Des Moines, IA	SPI	Springfield, IL
EYW	Key West, FL	SPS	Wichita Falls, TX
FAR	Fargo, ND	WAK	Wake Island, Pacific

Table 3

Anemometer Height Z_a, Mean Speed $\bar{V}$, and

Number of Years Averaged n, for Interannual Variability

Site	Z_a m	(ft)	$\bar{V}$ m/s	(MPH)	n	Site	Z_a m	(ft)	$\bar{V}$ m/s	(MPH)	n
ABR	6.10	(20)	5.01	(11.2)	12	FSD	5.18	(17)	5.05	(11.3)	15
ACT	7.01	(23)	4.83	(10.8)	12	GRI	6.10	(20)	5.23	(11.7)	15
AKN	6.10	(20)	4.78	(10.7)	14	GSW	6.71	(22)	4.87	(10.9)	15
AMA	7.01	(23)	6.26	(14.0)	15	GTF	6.71	(22)	5.36	(12.0)	12
BDR	25.60	(84)	5.63	(12.6)	15	HAT	9.75	(32)	5.14	(11.5)	18
BET	6.10	(20)	5.54	(12.4)	13	HLI	6.10	(20)	5.19	(11.6)	12
BIL	7.62	(25)	5.19	(11.6)	18	HNL	7.62	(25)	5.14	(11.5)	14
BOS	6.71	(22)	5.54	(12.4)	13	HOG	6.40	(21)	5.45	(12.2)	13
BTI	6.10	(20)	5.90	(13.2)	11	HON	6.10	(20)	5.14	(11.5)	14
BUF	6.10	(20)	5.01	(11.2)	17	JFK	6.10	(20)	5.45	(12.2)	14
CDB	6.40	(21)	7.51	(16.8)	14	LGA	6.10	(20)	5.19	(11.6)	14
CNK	6.40	(21)	5.45	(12.2)	14	MKE	6.10	(20)	5.01	(11.2)	18
CPR	6.10	(20)	5.72	(12.8)	12	MWN	12.50	(41)	15.11	(33.8)	10
CRP	7.01	(23)	5.41	(12.1)	16	RST	6.10	(20)	5.68	(12.7)	16
CYS	10.6	(33)	5.95	(13.3)	18	SDB	9.14	(30)	5.72	(12.8)	10
DDC	6.10	(20)	5.81	(13.0)	15	SGF	6.10	(20)	4.43	(9.9)	14
DLH	6.40	(21)	4.87	(10.9)	18	SNP	12.80	(42)	8.09	(18.1)	13
DSM	6.10	(20)	4.65	(10.4)	15	SPI	6.10	(20)	4.92	(11.0)	13
EYW	7.01	(23)	4.96	(11.1)	12	SPS	6.40	(21)	5.28	(11.8)	14
FAR	6.10	(20)	5.19	(11.6)	15	WAK	6.40	(21)	6.30	(14.1)	13

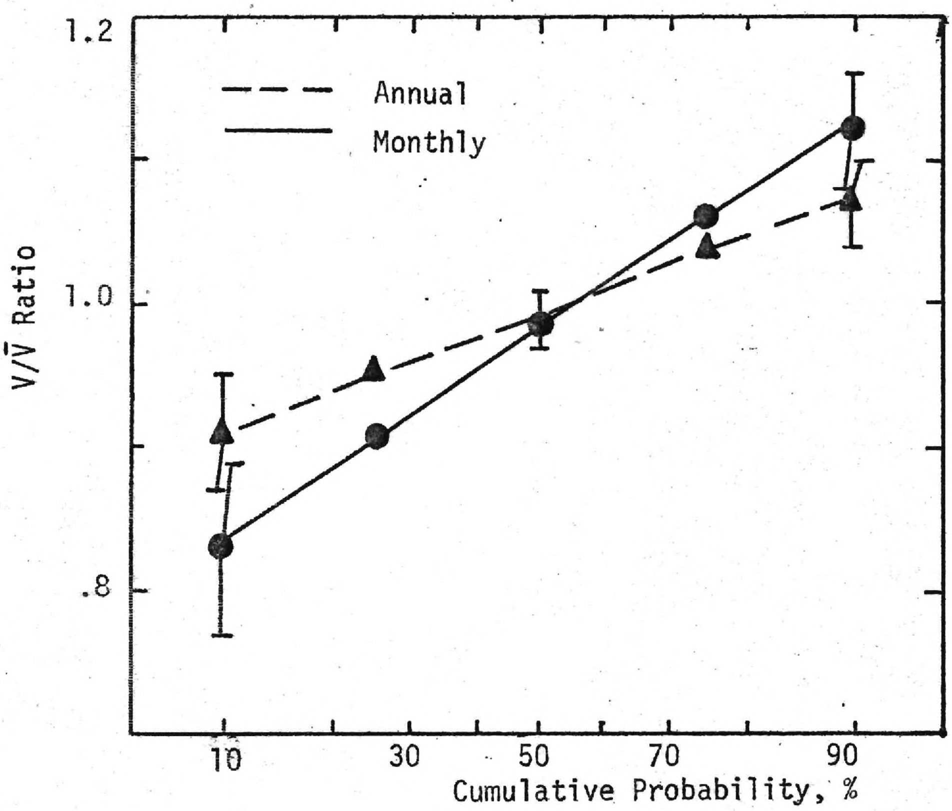


Figure 1 - Average Distribution of Interannual Wind Variability.

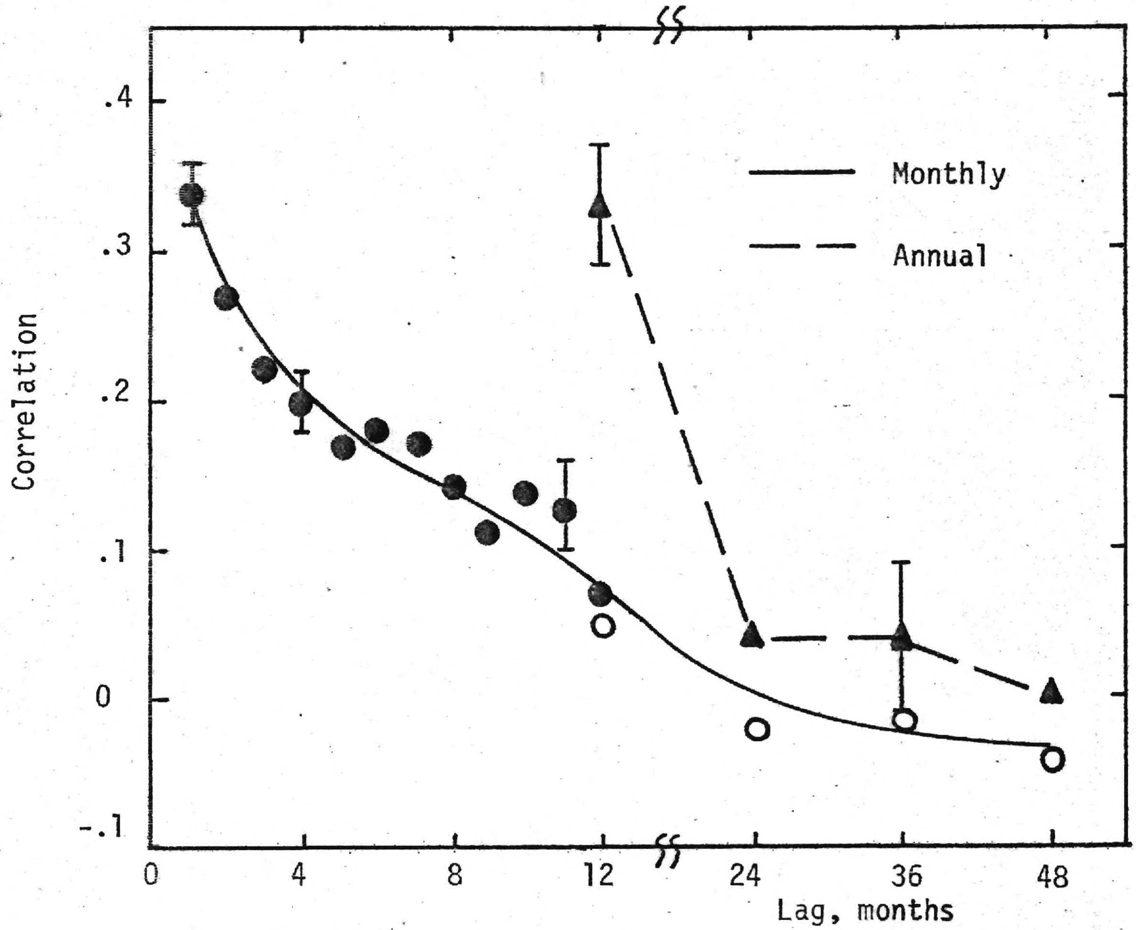
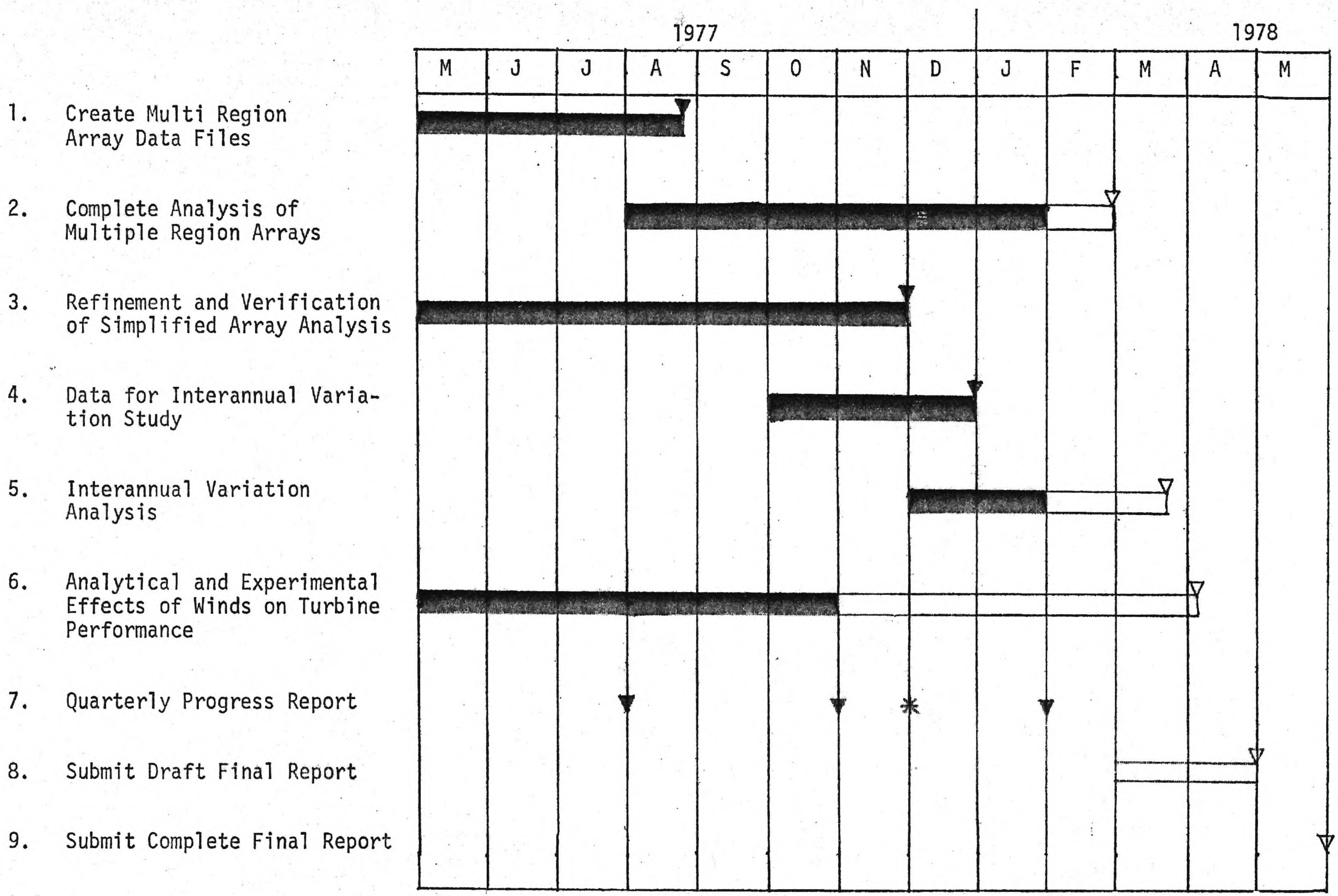


Figure 2 - Average Time Autocorrelation of Monthly or Annual Mean Wind.

Milestone Chart



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ANNUAL PROGRESS REPORT

PROJECT NO. E-16-617

ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

By

C. G. Justus and A. S. Mikhail

Prepared for

THE UNITED STATES DEPARTMENT OF ENERGY
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ENERGY STATISTICS FOR LARGE WIND TURBINE ARRAYS

Annual Progress Report

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EXECUTIVE SUMMARY

Results of earlier studies of large arrays of wind energy conversion systems (WECS) are summarized and synthesized into a methodology whereby distributions of array-averaged wind speed and distributions of power output can be calculated for arrays of any number of sites and any spatial size. Required input for the method consists of: (1) array mean wind speed, (2) maximum distance between sites within the array, (3) number of sites in the array, and (4) standard deviation (or Weibull scale factor), either measured or inferred from reference statistical data. Sample results using this methodology are presented along with sensitivity analyses of the various input parameters. Results are also presented for multiregional and continental arrays. These very large arrays were studied to determine the maximum benefit obtainable from the wind diversity effect. The cross-correlations, which fall off with array size more-or-less exponentially, are not appreciably negative at large separations. Since array diversity effects vary inversely with the spatial cross-correlation, negative cross-correlations would be desirable. The lack of negative-valued cross-correlations for large array sizes means that beyond a size of about 1500 km no additional benefit is gained from the array diversity effect.

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1. INTRODUCTION

Wind energy statistics of large arrays of wind energy conversion systems (WECS) have been studied in four regions of the country: the New England and Central United States Regions (Justus, 1976) and the Great Lakes and Pacific Coast Regions (Justus and Hargraves, 1977a). This report presents results obtained from multiregional arrays of selected sites from these four previously studied regions. The purpose of these multiregional analyses has been to investigate the upper limits of wind diversity benefit by studying large spatial areas to aid in determining if power might be produced by WECS in areas of high wind potential and transmitted over long distances to areas of high demand, and to use data from all four of the individual regions in developing and refining a generic methodology with which array statistical properties may be simulated adequately with simple averages or single site input information.

Table 1 shows the list of sites and their code designations for the sites used in the multiregional array studies. Figure 1 shows the locations of these sites. The multiregional array analysis results are presented in Appendices A through C. The main body of the report discusses the development and validation of the generic model for array simulation. Appendix D gives additional example applications of the generic model.

Recently Kahn (1978) has used the methodology discussed here and in the earlier array studies (Justus and Hargraves, 1977a) to evaluate capacity credit for WECS arrays dispersed over California. The methods discussed in this report were used in the JBF Scientific

Corporation-Georgia Tech-New England Gas and Electric Association (NEGEA) study of WECS (JBF, 1978) in the NEGEA utility service area. These methods were used for capacity credit analysis, loss of load probability analysis, optimum mix planning, feasible WECS penetration analysis, economic cost modeling analysis, and in assessment of the operating reserve impact of WECS (see these studies for details).

Table 1. Continental Array Sites

NORTHEAST		CENTRAL U. S.	
ACY	Atlantic City, NJ	AMA	Amarillo, TX
BOS	Boston, MA	CNK	Concordia, KS
BLU	Bluehill, MA	DDC	Dodge City, KS
LGA	New York (Laguardia), NY	GLD	Goodland, KS
PVD	Providence, RI	GRI	Grand Island, KS
		OKC	Oklahoma City, OK
		RSL	Russell, KS

GREAT LAKES		PACIFIC COAST	
BUF	Buffalo, NY	OTH	North Bend, OR
DTW	Detroit, MI	SDB	Sandberg, CA
ERI	Erie, PA	SFO	San Francisco, CA
MFD	Mansfield, OH	SMP	Stampede Pass, WA
MKE	Milwaukee, WI		
MKG	Muskegon, MI		

Figure 1. Map of the Continental Array Sites.

2. SUMMARY OF ARRAY CORRELATION RESULTS

Arrays of various spatial size have been studied in the Northeast and Central United States regions (Justus, 1976) and the Great Lakes and Pacific Coast regions (Justus and Hargraves, 1977a). A model described in the latter report has been developed which allows evaluation of the array power output distribution for any specified WECS operating characteristics, if input values for the mean wind speed, the standard deviation (or Weibull shape factor k), the number of sites in the array, and the average spatial cross-correlation for the array are provided. For generic application of this model, a convenient method is needed for estimating the average spatial cross-correlation ($\bar{\rho}$) for an array of arbitrary size. The results of the earlier array studies, plus the newer data from the multiregional arrays discussed in Appendices A through C, provide useful information concerning the relationship of $\bar{\rho}$ to array size, as characterized by the average separation of the array sites.

As an example of the concept of average separation of array sites (the important parameter determining average spatial correlation of the array), consider a simple linear array of n sites, each separated by the same distance (r), as shown in Figure 2. The maximum site separation (R) for this case is (n-1)r. For such a linear array there are n-1 site pairs with separation r, n-2 with separation 2r, etc. all the way to only one pair that has separation (n-1)r. The total number of site pairs is thus (n-1) + (n-2) + ... + 3 + 2 + 1. This sum is expressed mathematically as $\sum_{j=1}^{n-1} j$, where j is a dummy summation index.

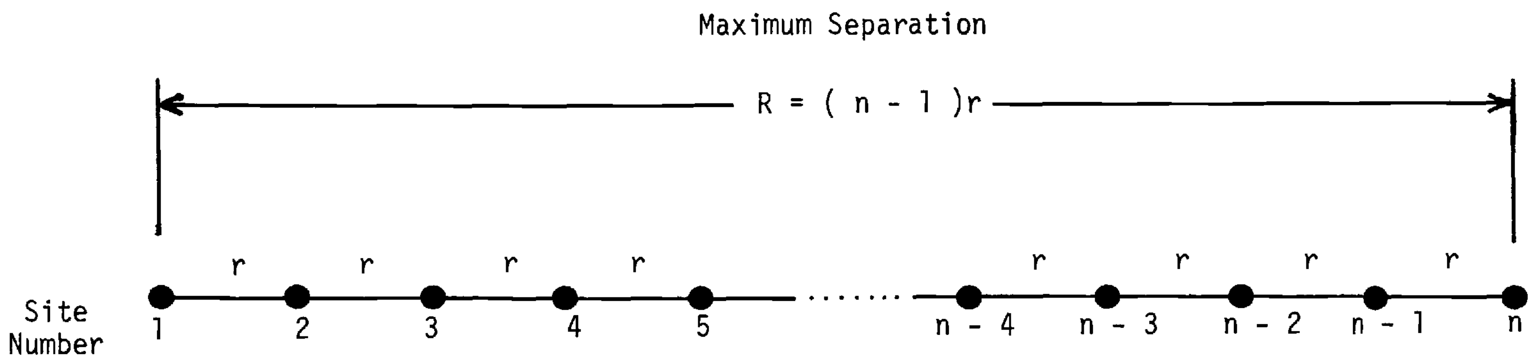

Figure 2. Illustration of Average Site Separation for Simple Linear Array of n Sites with Separation r.

From the algebra of sums of integers, this sum is equal to $n(n-1)/2$. The average site separation ($\bar{r}$) is found by the summation

$$\bar{r} = [(n-1)r + (n-2)2r + (n-3)3r + \ldots + 3(n-3)r + 2(n-2)r + (n-1)r]/[n(n-1)/2].$$

Again, by using the notation of sums of integers, this equation reduces to

$$\bar{r} = 2 \sum_{j=1}^{n-1} (n-j)jr/[n(n-1)],$$

which, by the algebra of sums of integers, can be simplified as follows:

$$\bar{r} = 2r[n \sum_{j=1}^{n-1} j^2]/[n(n-1)]$$

$$= 2r[n^2(n-1)/2 - n(n-1)(2n-1)/6]/[n(n-1)],$$

i.e.,

$$\bar{r} = (n+1)r/3 = (n+1)R/[3(n-1)]. \qquad (1)$$

Thus, if $n=2$, the average separation is just the separation between the one site pair, i.e. $\bar{r} = R = r$. But if n is large, the limiting value for $\bar{r}$ is $R/3$. Thus, for the simple linear array, provided n is reasonably large (e.g., $n \geq 20$), the average separation is 1/3 the maximum separation, independent of the number of sites.

Of course, there is no reason why the analysis of such a simple linear array should relate directly to the general case of a two dimensional, dispersed array with arbitrary orientation and spacing between site pairs. Unfortunately this general case cannot be solved analytically for the expected $R/\bar{r}$ ratio as could the case of the simple linear array. Therefore, the assumption is made that the linear array equation 1 establishes the trend of $\bar{r}$ versus number of sites (n), and that only the numerical constant changes for the general case. That is, in the general case the numerical value in the denominator is assumed to change from 3 to some other value, which can be determined empirically.

Table 2 summarizes all of the previous array study results for maximum separation of array site pairs (R) and average separation of array site pairs ($\bar{r}$), for the various number of sites (n) in each examined array. This table shows that the observed $R/\bar{r}$ ratio is, on the average, close to 88% of that expected from the simple linear array (average factor 0.88 $\pm$ 0.09). Thus, for an arbitrary array of maximum separation of site pairs R, equation 1 with the modified, empirically determined factor of 2.64 (=0.88 x 3) may be used instead of 3 in the denominator, namely

$$\bar{r} = (n+1)R/[2.64(n-1)]. \qquad (2)$$

Table 2 also gives values for the average spatial cross-correlation of the arrays studied earlier. The average cross-correlation ($\bar{\rho}$) for the array is the intersite pair correlation averaged over all of the independent site pair combinations. From equation B-2 the average correlation ($\bar{\rho}$) is given by

$$\bar{\rho} = 2_i\Sigma\Sigma_j\rho_{ij}/n(n-1), \qquad (3)$$

where the spatial correlation ρ_{ij} is the correlation of the deviations of simultaneously-observed, one-minute average wind speeds from the respective monthly average wind speeds at the sites i and j of the pair whose correlation is being evaluated. The correlation data of Table 2 are shown graphically in Figure 3. The annual correlation curve is approximated reasonably well, for distances of 200 km and

Table 2. Summary of Array Separation and Spatial Cross-Correlation Results*

Array	Maximum Separation R, km	Average Separation $\bar{r}$, km	Number of Sites n	Theoretical Ratio $\frac{3(n-1)\bar{r}}{(n+1)R}$	Average Cross-Correlation, $\bar{\rho}$ summer	winter	annual
Mid Atlantic	231	111	8	0.89	0.53	0.69	0.63
New England Coastal	365	198	7	0.82	0.48	0.63	0.56
E. Great Lakes	537	246	14	0.84	0.48	0.52	0.53
New England Inland	671	284	13	0.92	0.47	0.54	0.51
New England	987	309	28	1.14	0.44	0.55	0.50
Great Lakes	793	325	24	0.88	0.45	0.52	0.48
North Central	857	374	13	0.89	0.35	0.40	0.41
South Central	775	389	12	0.78	0.36	0.42	0.40
Pacific Coast	1442	582	14	0.95	0.24	0.15	0.19
Central	1443	618	25	0.84	0.29	0.31	0.32
E. Coast & Gt. Lakes	1483	683	11	0.87	0.31	0.29	0.30
Gt. Lakes & Central	2192	1000	13	0.85	0.22	0.20	0.22
Central & Pacific	2467	1287	11	0.97	0.17	0.19	0.19
Continental	4676	1845	22	0.92	0.16	0.11	0.13

* All averages are for 3 to 5 years, except for E. Great Lakes array which is a one year average.

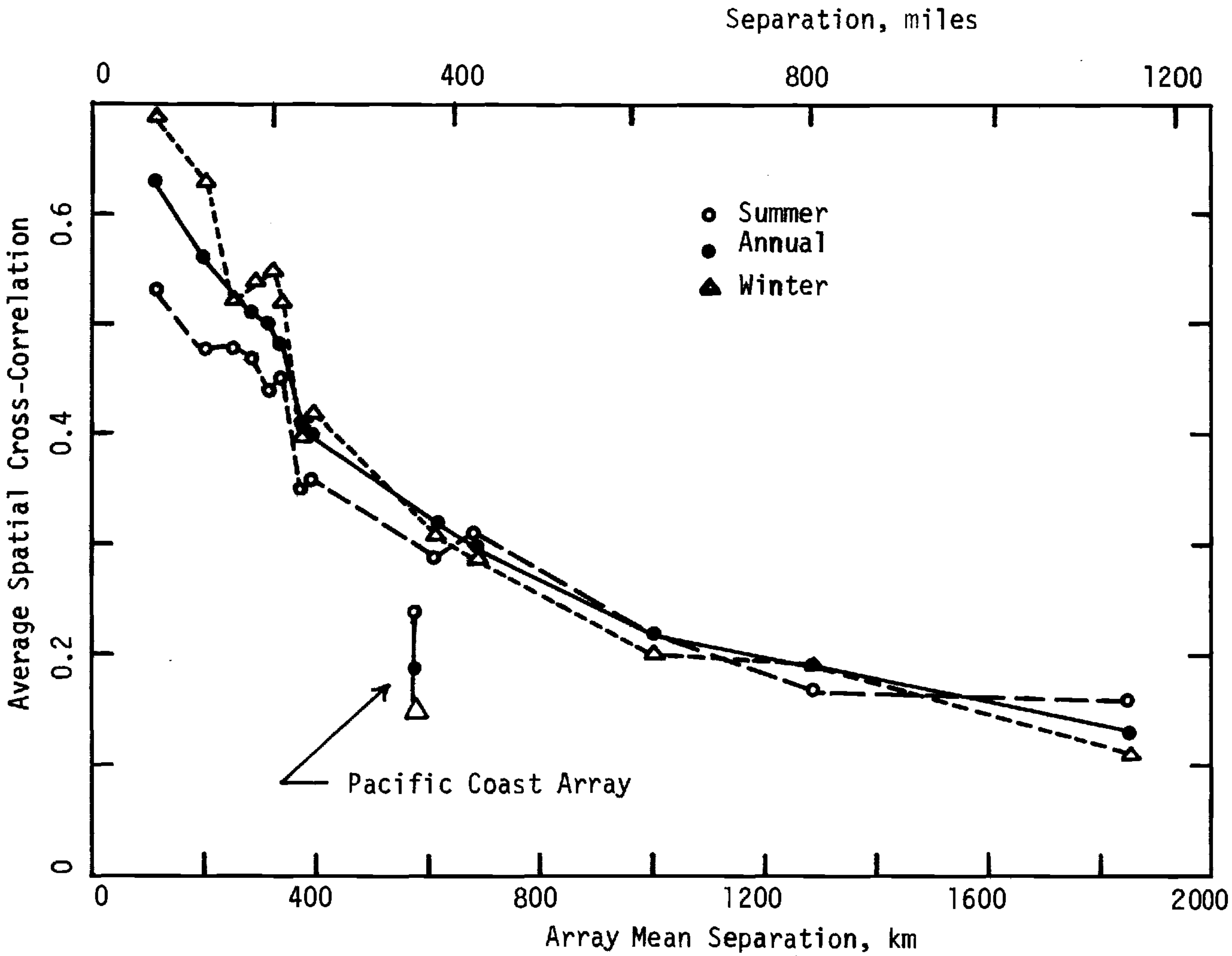

Figure 3. Average Array Spatial Correlation Versus Average Array Site Separation Distance.

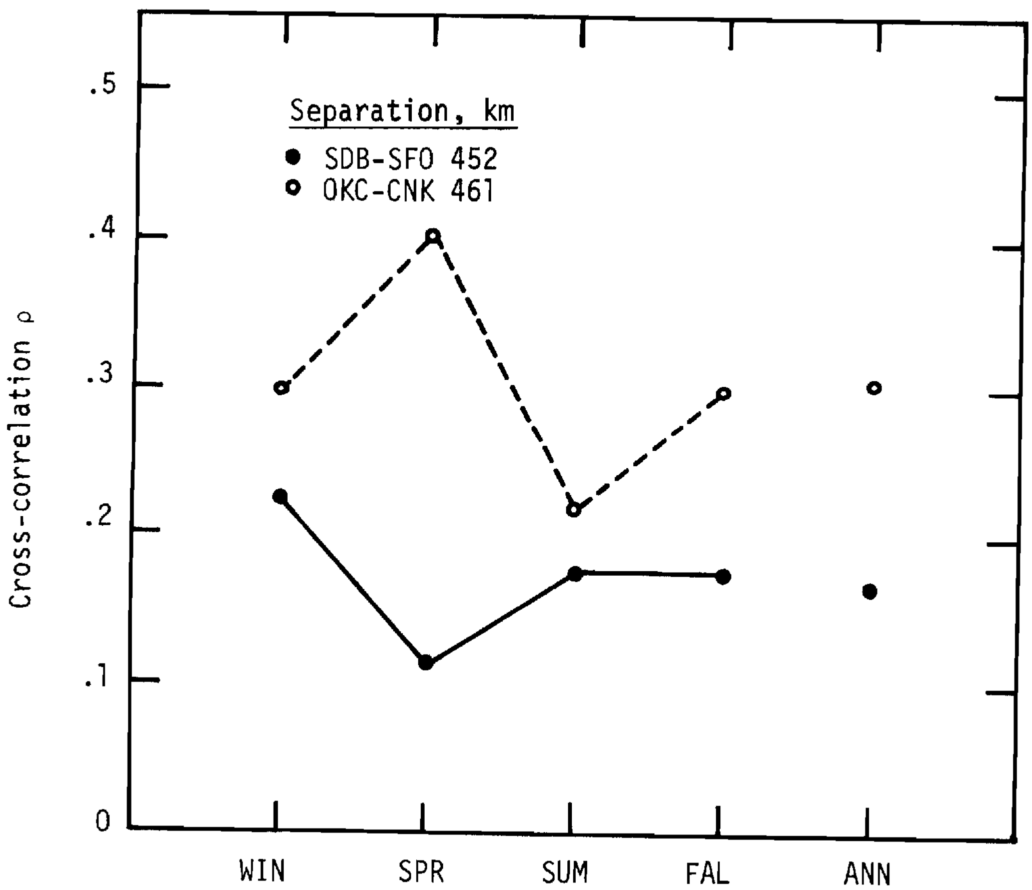

Figure 4. **Comparison of Spatial Correlations between Pacific Coast and Central United States Site Pairs.** SDB = Sandberg, California; SFO = San Francisco; OKC = Oklahoma City; CNK = Concordia, Kansas.

greater, by the following relation:

$$\bar{\rho}(\bar{r}) = \exp[-(\bar{r}/520)^{0.57}], \tag{4}$$

for $\bar{r}$ in km.

The Pacific Coast array correlation in Figure 3 is anomalously low, probably due in part to the effects of terrain on the Pacific Coast region correlations. See for example Figure 4, which compares correlations between two Pacific Coast sites (SDB-SFO) and two Central United States sites (OKC-CNK) of comparable separation and latitude difference. Sandberg (SDB) is a mountain top site, while San Francisco (SFO) is in a flat coastal zone on the other side of the coastal range from Sandberg. In contrast both Oklahoma City (OKC) and Concordia, Kansas (CNK) are in flat terrain. It is reasonable, therefore, to assume that the SDB-SFO cross-correlation is lower than that for OKC-CNK because of the terrain influence on the former site pair.

The Pacific Coast array cross correlation averages about 55% of the value of the correlation at the same separation for the other arrays studied. Therefore, for arrays in mountainous terrain the use of equation 4 may significantly overestimate the actual average correlation of the array. Of course, for a given application, it would be best to do sample cross-correlations for some sites in the actual array terrain being studied, instead of relying on the average equation 4.

3. THE SIMPLE ARRAY STATISTICS MODEL

This section describes the validation and example results of the simple array statistics model when actual array mean speed, root-mean-square (rms) standard deviation in speed, mean site separation, and mean cross-correlation data are available. Section 4 gives specific examples for cases when only mean wind speed and array size are known and the other parameters are inferred from the correlation results of Section 2 and the revised reference statistics (Figure D-5 in Appendix D).

The development of the array statistics model was described by Justus and Hargraves, (1977a) and is elaborated in Appendices B and C of this report. The necessary data for the array wind speed distribution model consist of: 1) n, the number of sites (e.g. individual WECS farms) in the array, 2) the average cross-correlation ($\bar{\rho}$) for the array (average observed data from equation 3 or estimated from Figure 3 or equation 4), 3) average wind speed ($\bar{V}_n$) of the array, which can be taken as either observed averages ($\bar{V}_0$) from equations 6 or 11, or average speed from a single site ($\bar{V}_1$), representative of the array, and 4) the rms average standard deviation in wind speed (σ_0 from equations 7 or 12, or standard deviation (σ_1) for the single representative site). If, instead of standard deviation data, Weibull shape parameters (k), for average array distribution or representative single site distributions, are known, then these data can be used instead of σ_0 or σ_1. The following examples illustrate the steps for application of this model.

The key step in modeling the wind speed distribution for the array is the conversion from the larger standard deviation for distributions of wind speed at a single site to the smaller standard deviation for the distribution of wind speeds averaged across the array. The wind speed, at a given time, averaged across the array, is called the "array-average wind speed" for that time. A smaller standard deviation and a narrower distribution for the array-average wind speed result because of the smoothing effect of the averaging across the array. Both the individual site, or rms average site, and the array-average wind speed distributions are modeled as Weibull functions. Therefore, the standard deviation conversion can be accomplished alternately via a Weibull shape factor conversion.

For evaluation of the distribution of power output from the array, a model for array power versus array-average wind speed is needed, such as that provided by equation 16. Equation 16, applied with equations 15 and 17, allows the probability of array power being within any desired limits to be evaluated.

Mathematical Description of the Model

Consider an array of n wind energy sites. Let each site have mean wind speed at hub height $\bar{V}_i$, with standard deviation σ_i. At a given time (t) the average wind speed across the array, $V_n(t)$, is the average of all of the wind speeds at that time, i.e.,

$$V_n(t) = n^{-1} \sum_{i=1}^{n} V_i(t). \qquad (5)$$

Since this averaging process is linear, the time mean of the array-average wind speed ($\bar{V}_n$) is the same as the average of the individual site mean winds ($\bar{V}_0$), i.e.,

$$\bar{V}_n = T^{-1} \sum_{t=1}^{T} V_n(t) = \bar{V}_o = n^{-1} \sum_{i=1}^{n} \bar{V}_i. \tag{6}$$

The standard deviation (σ_n) of the array average speeds, $V_n(t)$, is, in general, a complicated function of individual site standard deviations (σ_i), and cross product terms such as $\sigma_i\ \sigma_j\ \rho_{ij}$, where ρ_{ij} is the spatial cross-correlation between site i and j. For the simpler case in which all sites have the same mean speed ($\bar{V}_i = \bar{V}_j = \bar{V}_o$; all i, j) and the same standard deviation ($\sigma_i = \sigma_j = \sigma_o$; all i, j), the standard deviation of the array-average wind speed, $V_n(t)$, about the array mean speed ($\bar{V}_n$) is

$$\sigma_n = \sigma_o\ \{[1 + (n-1)\bar{\rho}]/n\}^{1/2}, \tag{7}$$

where $\bar{\rho}$ is the average cross-correlation,

$$\bar{\rho} = \sum_{ij}\sum \rho_{ij}/N, \tag{8}$$

and N is the number of independent site pairs N = n(n-1)/2 (see Appendix B, equation B-2). The actual ratio of σ_n/σ_o would not be expected to correspond to the theoretical ratio $(\sigma_n/\sigma_o) = \{[1+(n-1)\bar{\rho}]/n\}^{1/2}$ given by equation 7, because the actual arrays do not have equal means and standard deviations. However, if the array rms standard deviation is used, as evaluated by

$$\sigma_o = [\sum_{i=1}^{n} \sigma_i^2/n]^{1/2}, \tag{9}$$

then the data in Table 3, with n and $\bar{\rho}$ values from Table 2, show that there is a close similarity between observed array σ_n/σ_o values and the theoretical relation of equation 7. Figure 5 shows a linear regression plot of observed σ_n/σ_o values versus theoretical values. The proportional relation (no constant term) which provides the best fit to these

Table 3. Standard Deviations for rms Single Site (σ_o), for Array-Average Wind Speed σ_n, and the Observed Ratio σ_n/σ_o, Versus Theoretical Ratio $\{[1 + (n-1)\bar{\rho}]/n\}^{\frac{1}{2}}$

Array	Standard Deviation of Single Site σ_o	Standard Deviation of Array σ_n	Observed Ratio σ_n/σ_o	Theoretical Ratio $\{[1 + (n-1)\bar{\rho}]/n\}^{\frac{1}{2}}$
Mid Atlantic	2.9	2.7	0.93	0.82
New England Coastal	3.3	2.8	0.85	0.79
New England Inland	3.2	2.6	0.83	0.74
New England	3.2	2.4	0.81	0.72
Great Lakes	3.0	2.3	0.77	0.71
North Central	3.2	2.4	0.75	0.67
South Central	3.0	2.2	0.75	0.67
Pacific Coast	3.0	1.6	0.53	0.50
Central	3.1	2.0	0.65	0.59
E. Coast & Gt. Lakes	3.1	2.1	0.68	0.60
Gt. Lakes & Central	3.1	1.8	0.58	0.53
Central & Pacific	3.2	1.8	0.56	0.51
Continental	3.2	1.5	0.47	0.41

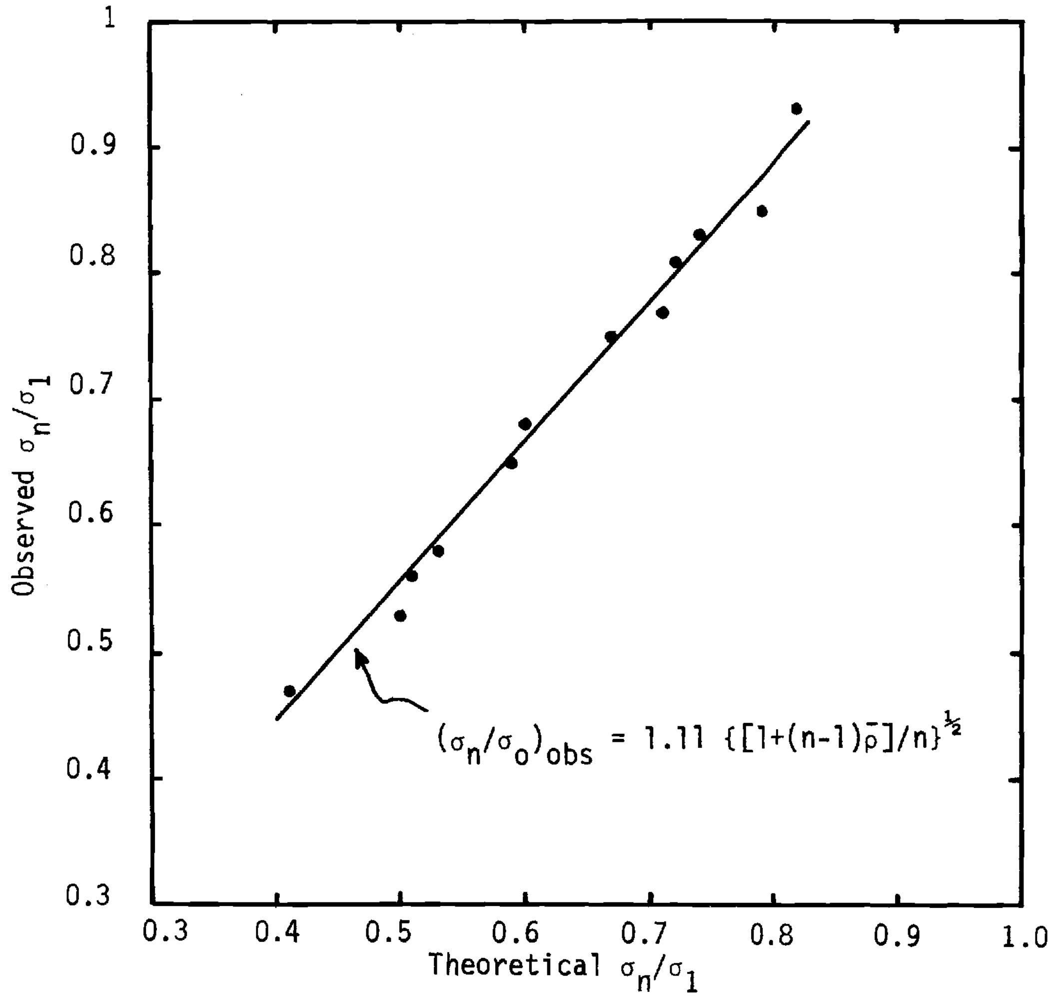

Figure 5. Linear Regression of Observed Array-to-Single-Site Standard Deviation Ratio σ_n/σ_o Versus the Theoretical Ratio $\{[1 + (n-1)\bar{\rho}]/n\}^{\frac{1}{2}}$. Data from Table 3.

data is found to be

$$\sigma_n/\sigma_o = 1.11 \{[1 + (n-1)\bar{\rho}]/n\}^{\frac{1}{2}}. \tag{10}$$

This regression explains approximately 97% of the variance among the observed σ_n/σ_o values.

The equations 6 and 9 presume equal weighting of the sites in the average, that is, equal numbers of WECS units at all sites. For the case in which there are different numbers of WECS units (m_i) at the n sites, with total number $M = \Sigma m_i$, then $\bar{V}_o$ and σ_o can be evaluated by weighted averaging

$$\bar{V}_o = M^{-1} \sum_{i=1}^{n} m_i \bar{V}_i \tag{11}$$

$$\sigma_o = [\sum_{i=1}^{n} m_i \sigma_i^2/M]^{\frac{1}{2}} \tag{12}$$

[see equations C-3 and C-4].

With the standard deviation transformation accomplished via equation 10, the $\bar{V}_n$ and σ_n values can be used to determine the Weibull distribution parameters c_n and k_n for the array-average wind speed. Namely

$$k_n = (\sigma_n/\bar{V}_n)^{-1.086} = (1.11\ \sigma_o/\bar{V}_o)^{-1.086}\{[1+(n-1)\bar{\rho}]/n\}^{-0.543} \tag{13}$$

$$c_n = \bar{V}_o/\Gamma(1+1/k_n) \tag{14}$$

[see equations C-5 and C-6]. With these k_n and c_n values, the probability of finding array speeds (V_n) between any limits, say V_j and V_k, is found by

$$p(V_j \leq V_n \leq V_k) = \exp[-(V_j/c_n)^{k_n}] - \exp[-(V_k/c_n)^{k_n}]. \tag{15}$$

Examples with Average Array Site Data

The Great Lakes and Pacific Coast arrays (Justus and Hargraves, 1977a) had observed annual average wind speed ($\bar{V}_n$) of 7.26 m/s at hub-height, computed from equation 6 with individual site annual mean speeds $\bar{V}_i$, and array rms standard deviation σ_0 of 2.97 m/s, computed from equation 4 with standard deviations of individual sites σ_i, as given in Tables 4 and 5. The average spatial correlations were 0.48 for the Great Lakes (n=24) array and 0.19 for the Pacific Coast (n=14) array. Tables 4 and 5 also show the observed wind speed distributions for the average single site and the observed distribution of array-average wind speeds (speed averaged across the array at a given time). The model distributions of array-average wind speed also given in Tables 4 and 5, were calculated as follows: 1) parameters k_n and c_n for the Weibull distribution of array wind speeds were calculated from $\bar{V}_0$, σ_0, $\bar{\rho}$, and n by equations 13 and 14, 2) distributions $p(V_i \leq V_{array} \leq V_k)$ for array-average wind speed were evaluated from k_n and c_n via equation 15. Values for σ_n, k_n, and c_n for each array are given in Tables 3 and 4. Figures 6 and 7 give a graphical comparison between the wind speed distributions observed at the average single site, observed for the array, and calculated for the model array data from Tables 4 and 5. Note that the array distributions are narrower for both cases (fewer occurrences of high and low extremes) than the single site distributions, because of the effects of array averaging.

Examples with Single Site Data

For what would be the severest test of the ability of single site data to represent an array, the continental array case was selected for

Table 4. Observed and Model Array Wind Speed Distributions for the Great Lakes Region Array (average annual distribution)*

Hub Height Winds V_i, V_j (m/s)	Observed Avg. Single Site Distribution $p(V_i \leq V \leq V_j)$, %	Observed Array Distribution $p(V_i \leq V \leq V_j)$, %	Calculated Array Distribution $p(V_i \leq V \leq V_j)$, %
0-4	15.1	6.5	8.6
4-6	18.7	24.1	21.7
6-8	25.6	34.4	31.7
8-10	20.7	22.8	25.5
10-12	12.5	9.4	10.4
12-14	5.3	2.1	1.9
14-16	1.3	0.5	0.1
> 16	0.7	0.6	0.0

* The following parameter values were used to evaluate the distributions in this table:

$\bar{V}_0 = \bar{V}_n = 7.26$ m/s

$\sigma_0 = 2.97$ m/s　　$\sigma_n = 2.34$ m/s

$\bar{\rho} = 0.48$　　$k_n = 3.43$

$n = 24$　　$c_n = 8.08$ m/s

Table 5. Observed and Model Array Wind Speed Distribution for the Pacific Coast Region Array (average annual distribution)*

Hub Height Winds V_i, V_j (m/s)	Observed Avg. Single Site Distribution $p(V_i \leq V \leq V_j)$, %	Observed Array Distribution $p(V_i \leq V \leq V_j)$, %	Calculated Array Distribution $p(V_i \leq V \leq V_j)$, %
0-4	21.7	2.8	6.9
4-6	23.6	36.2	28.4
6-8	23.5	41.7	43.8
8-10	16.6	17.2	19.5
10-12	8.5	1.9	1.5
12-14	3.8	0.0	0.0
14-16	1.5	0.0	0.0
> 16	0.7	0.0	0.0

* The following parameter values were used to evaluate the distributions in this table:

$\bar{V}_0 = \bar{V}_n = 6.59$ m/s

$\sigma_0 = 3.02$ m/s $\qquad \sigma_n = 1.67$ m/s

$\bar{\rho} = 0.19$ $\qquad k_n = 4.44$

$n = 14$ $\qquad c_n = 7.24$ m/s

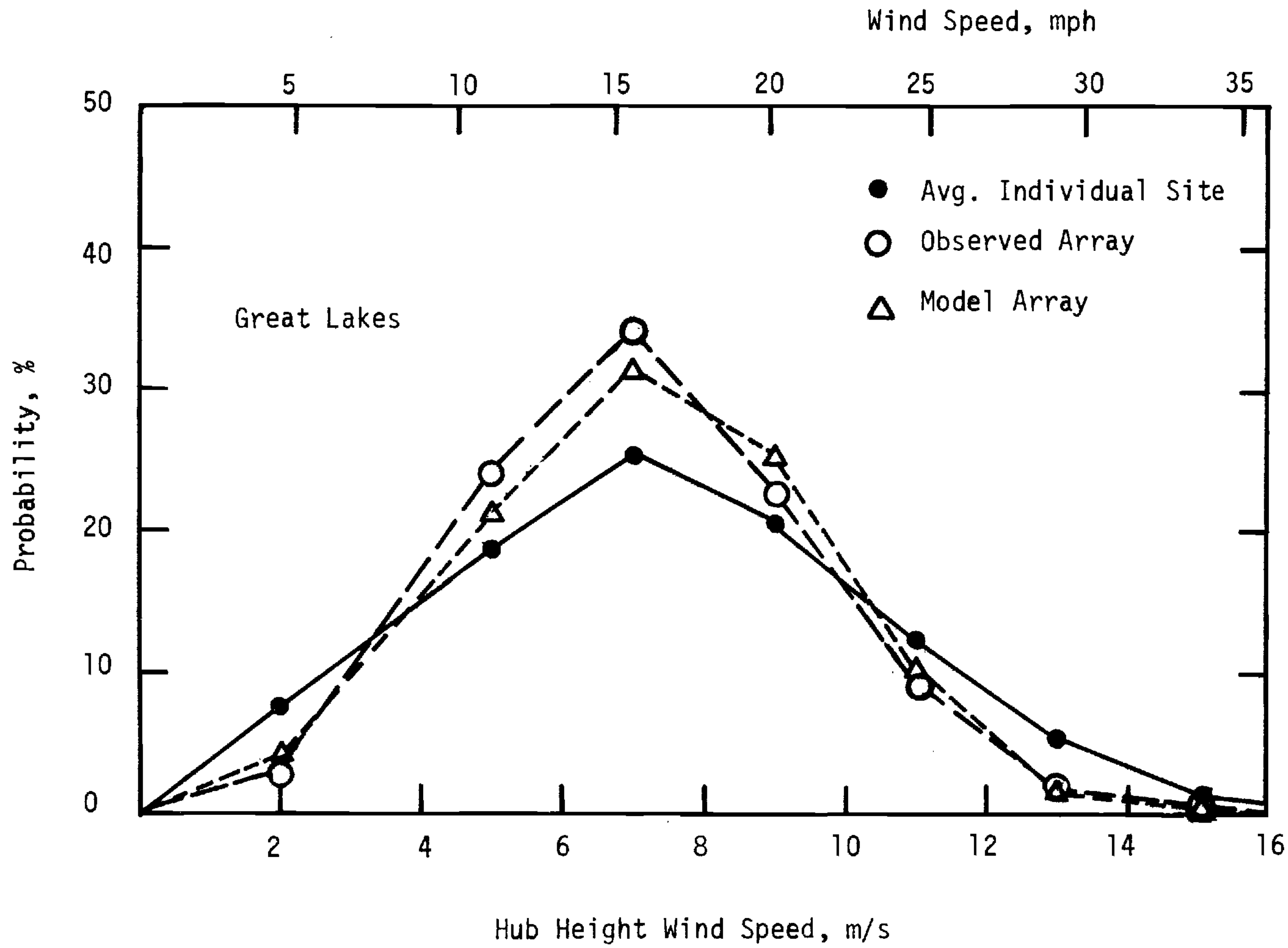

Figure 6. Observed and Model Array Wind Speed Distributions for the Great Lakes Array (Annual Average).

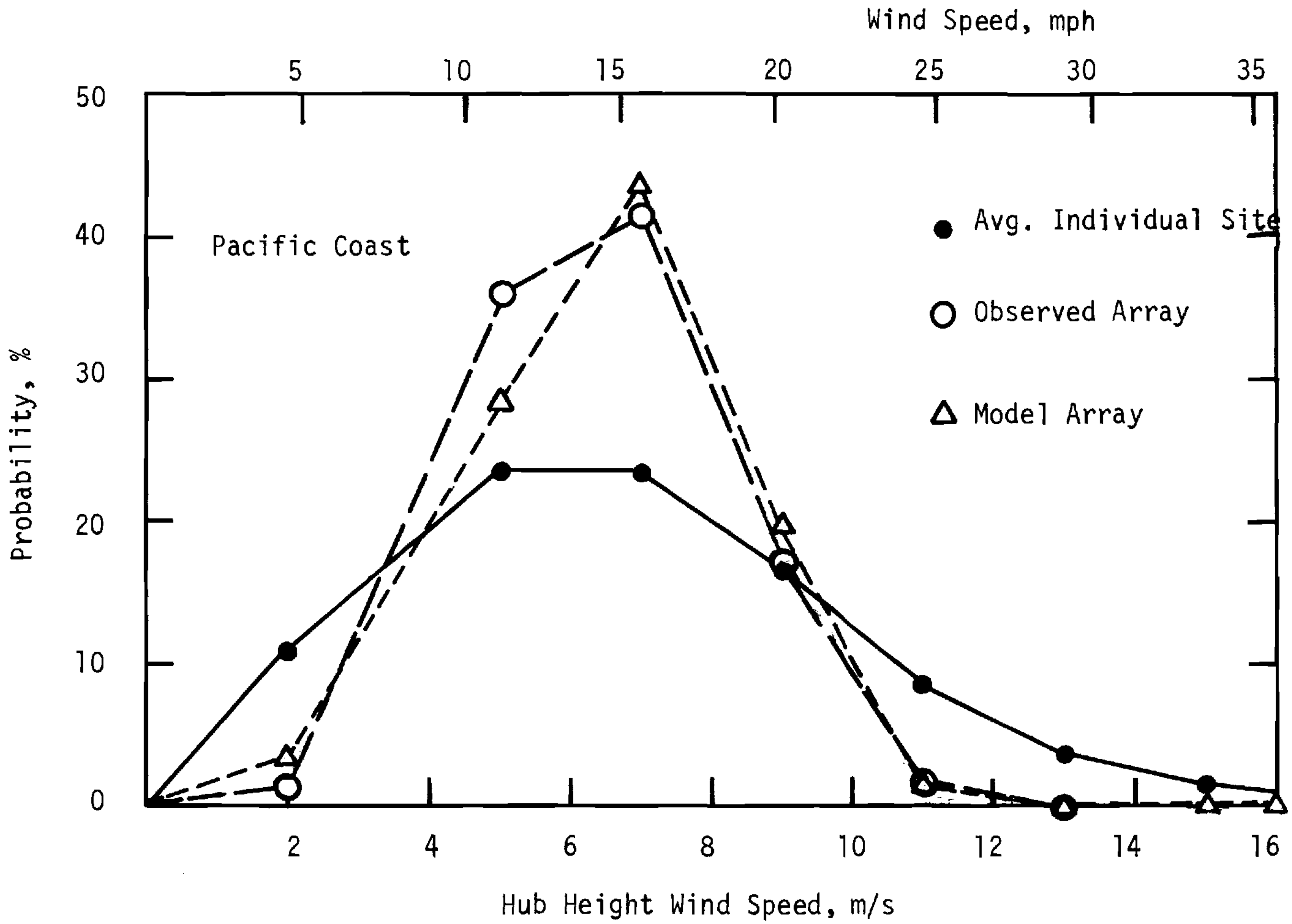

Figure 7. Observed and Model Array Wind Speed Distribution for the Pacific Coast Region (Annual Average).

model evaluation. This test is considered the severest because it involves the largest transformation of standard deviations from single site to array distribution. If the model works well in this "severest case" test, presumably it will work as well, or better, in less severe cases. From Table A-1, the annual array-average wind speed was 8.2 m/s. Milwaukee (MKE) was selected for the "representative" single site on the basis of the similarity of its annual average wind speed, $\bar{V}_1 = 8.2$ m/s at hub-height. MKE also had a standard deviation of $\sigma_1 = 3.2$ m/s, which was close to that for the rms average standard deviation for the array ($\sigma_0 = 3.1$ m/s). Close standard deviations are not entirely fortuitous, since σ tends to vary with $\bar{V}$ [$\sigma \approx K\bar{V}^{\frac{1}{2}}$].

The Continental array data were run for 1971, along with the single site model calculations for the same year. These results are shown and compared in Table 6 and Figure 8.

For the MKE model array distribution, the observed 1971 average wind speed at hub-height ($\bar{V}_1 = 8.05$ m/s) and standard deviation ($\sigma_1 = 3.11$ m/s) were used instead of σ_0 and V_0 in equations 13 and 14 to compute array distribution Weibull parameters k_n and c_n. These values were then used in equation 15 to compute the distribution.

To test the ability of single sites to adequately model arrays on the basis of matching average wind speeds only, the highest (BLU) and lowest (OTH) average wind speed sites were run through the array simulation program with their speeds adjusted so as to average that for the array. Average speeds before adjustment were: for BLU 10.7 m/s, and for OTH 7.0 m/s (1971 data). Since the standard deviation varies approximately as $\sigma \approx K\bar{V}^{\frac{1}{2}}$, this adjustment should yield σ_1 values for BLU and OTH, which are not as representative as for MKE. Adjusted

Table 6. Observed and Single Site Modeled Array Wind Speed Distributions for the Continental Array (annual average, 1971 data)*

Hub Height Winds V_j, V_k (m/s)	Observed Avg. Single Site Distribution $p(V_j \leq V \leq V_k)$, %	Observed Array Distribution $p(V_j \leq V \leq V_k)$, %	MKE Calculated Array Distribution $p(V_j \leq V \leq V_k)$, %
0-4	9.7	0.0	0.7
4-6	15.8	6.4	8.4
6-8	24.9	43.4	36.7
8-10	24.5	40.1	46.7
10-12	13.0	9.3	7.4
12-14	7.2	0.8	0.0
14-16	3.1	0.0	0.0
> 16	1.8	0.0	0.0

* The following parameter values were used to evaluate the distributions in this table.

	MKE	OTH (adjusted)	BLU (adjusted)
$\bar{V}_0$ = 8.08 m/s	$\bar{V}_1$ = 8.05 m/s	8.05	8.09
σ_0 = 3.14 m/s	σ_1 = 3.16 m/s	3.82	2.95
$\bar{\rho}$ = 0.13	σ_n = 1.44 m/s	1.75	1.35
n = 22	k_n = 6.46	5.26	7.00
	c_n = 8.63 m/s	8.74	8.64
	rms distribution error % = 4.0	4.7	4.9

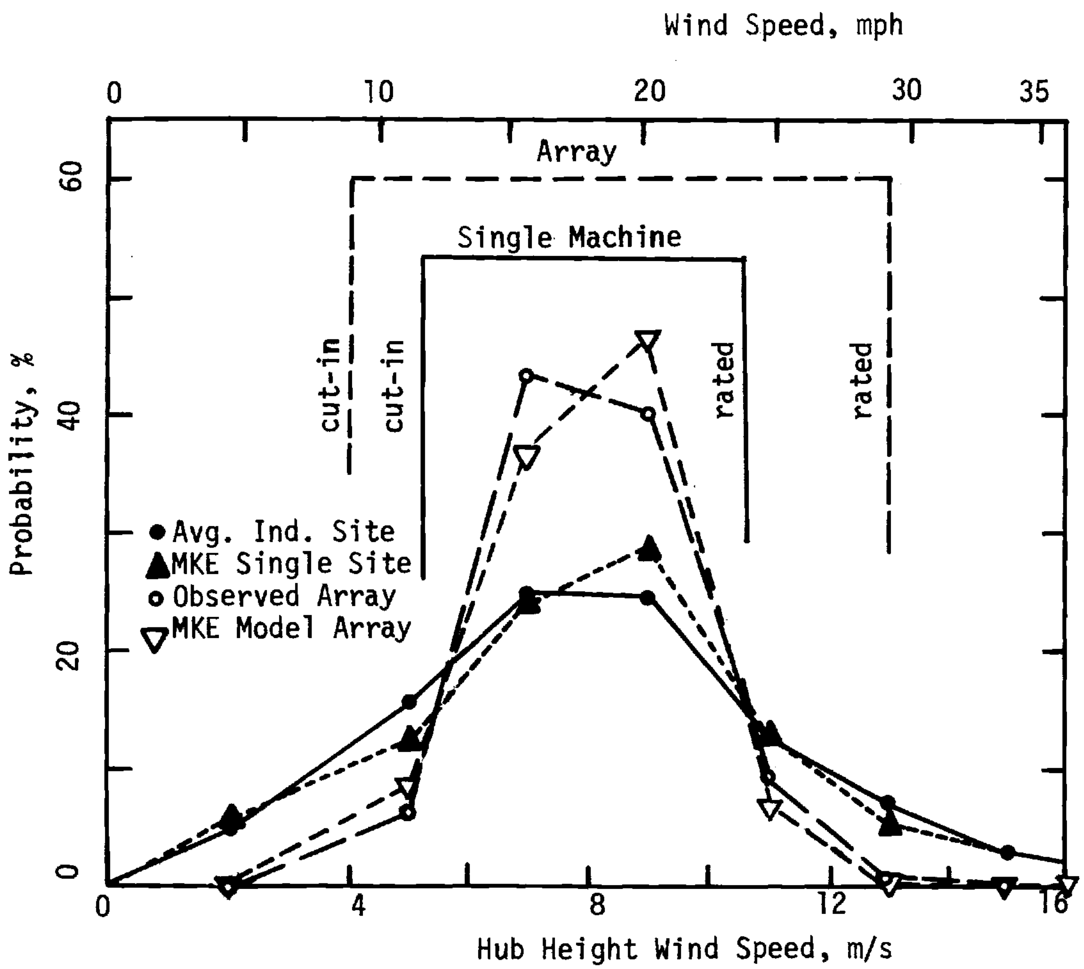

Figure 8. Observed and Single Site Modeled Array Wind Speed Distributions for the Continental Array (1971 Annual Average).

values are shown in Table 6. As a relative measure of "goodness of fit" of the respective model curves the rms deviation between observed and calculated distributions (0-14 m/s speed interval) was used. The rms deviation between the MKE model array and observed array distributions (0-14 m/s) was found to be 4.0%. The rms deviations between model array and observed array distributions (0-14 m/s) for BLU and OTH adjusted data were 4.7% and 4.9% respectively, only slightly higher than that obtained from the MKE model array. As a result, a single "representative" site for array modeling can be selected primarily on the basis of its average wind speed. (For sensitivity analysis of the effects of variation in mean speed $\bar{V}_n$, standard deviation σ_n, correlation $\bar{\rho}$, and other factors, see the "Array Model Sensitivity Analysis" section of this chapter).

Only the annual distribution was computed and the "representative" site was selected on the basis of annual average wind speed only. If details of wind speed distributions by season or by hour are desired, a site should be selected on the basis of seasonal and diurnal wind speed variation compatible to those expected for the array.

Array Power Output Modeling

To simulate the probability distribution of array power output, the array wind speed distribution must first be simulated, as above. Next a model for wind power output versus array wind speed, such as equation C-7

$$P(V_n)/P_r = a + b(V_n/V_r) \qquad (16)$$

must be used in conjunction with equation 15 for the power distribution calculations. This is done by inverting equation 16 to find the array wind speed, say V_j, which corresponds to the desired power $P(V_j) = P_j$, i.e.,

$$V_j = [(P_j/P_r) - a]V_r/b. \tag{17}$$

For a physical understanding of this procedure, the wind speed distribution shown in Figure 8 should be examined. For an individual wind turbine at an individual site, the probability distribution would be as shown by the solid curve in Figure 8. The single site cut-in and rated speeds are illustrated by the inner bracket labeled cut-in, rated single machine (5.3 m/s cut-in and 10.6 m/s rated for the example 2 MW wind turbine). Figure 8 shows that a single wind turbine at an individual site would be at zero power (wind speed below cut-in) or at full rated power (wind speed above rated) for a relatively large fraction of the time. In contrast, the distribution of array-average wind speeds (speeds averaged across the array at a given time) would be as shown in Figure 8 by the dashed curve with open circle data points. The array output power as a function of array speed has lower effective cut-in and higher effective rated speeds [equation D-10] than the individual machine. Even at low array average speeds at a given time, some of the individual sites have speeds above the machine cut-in; hence, some of the array sites are producing power (i.e., the array power is greater than zero). Similarly, even at high array-average wind speeds at a given time, some of the sites in the array have speeds below the machine rated speed; hence, not all sites in the array are producing power (i.e., the array power is less than rated). The combination of the narrower wind speed distribution with the broader effective range between cut-in and rated, means that the probability of power extremes (zero or full rated) are both significantly lower in the array case than for an individual machine.

Table 7 and Figure 9 are examples of the array power output distribution modeled from the MKE single site data given in Table 6. Model values are compared with observed probability distributions of power from the Continental array and observed power distributions from the average single site. The linear regression parameters for array power for the 2MW example machine [a and b for equation 17] can be computed by

$$a = \alpha_0 + \alpha_1(V_{in}/V_r) + \alpha_2(C_{pr}/C_{pm}) \tag{18}$$

$$b = \alpha_3$$

[see equation D-9]. The observed linear regression values for the Continental array ($a = -0.44$, $b = 1.17$) agree (to 2 decimal places) with the values computed from 18 with the low variance reference statistics α values (see Table 8). Array power distribution MKE model values are computed from equation 15, with array speed values V_j and V_k corresponding to array powers P_j and P_k computed from equation 17. As shown in Table 7, the rms deviation (0-1.8 MW interval) between observed and MKE model array distributions is 2.2%. This is used arbitrarily as a relative measure of the "goodness-of-fit" of the MKE model array distribution, when compared to other distributions in the sensitivity analysis.

Array Model Sensitivity Analysis

In order to test the sensitivity of the array model to its various input parameters the evaluations shown in Table 9 and Figures 10-13 were done. Compared to the reference case (MKE model of Continental

Table 7. Observed and Single Site Modeled Array Power Distributions for the Continental Array (1971 annual average)*

Array Powers per Unit P_j , P_k (MW)	Observed Array Power Probability (%)	MKE Model Array Probability (%)	
0	0.0	0.7	
0-0.2	0.2	1.9	
0.2-0.4	5.2	4.8	
0.4-0.6	12.6	10.3	
0.6-0.8	21.0	17.9	
0.8-1.0	20.4	24.0	
1.0-1.2	19.6	22.8	
1.2-1.4	12.9	13.2	
1.4-1.6	6.4	3.9	
1.6-1.8	1.7	0.4	
1.8-2.0	0.0	0.0	rms deviation 2.2%
2	0.0	0.0	

* The following parameter values were used to evaluate the distributions in this table.

P_r = 2MW $\quad \bar{V}_0$ = 8.05 m/s $\quad \sigma_1$ = 3.16 m/s

V_{in} = 5.3 m/s $\quad \sigma_0$ = 3.14 m/s $\quad \sigma_n$ = 1.44 m/s

V_r = 10.6 m/s $\quad \bar{\rho}$ = 0.13 $\quad k_n$ = 6.46

a = -0.44

b = 1.17 $\quad$ n = 22 $\quad c_n$ = 8.63 m/s

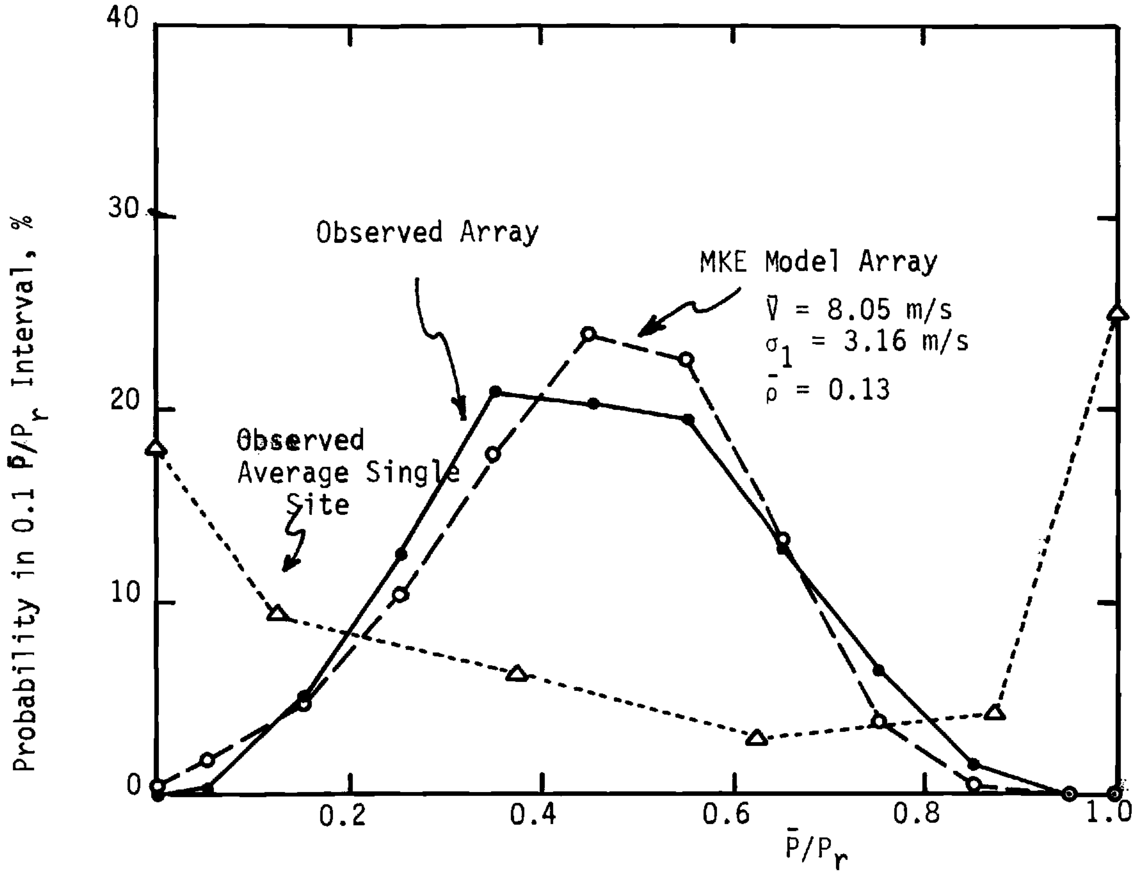

Figure 9. Observed and Single Site Modeled Array Power Distribution for the Continental Array (1971 Annual Average)

array, Table 7, also shown as column 1 in Table 9), array distributions with different values of $\bar{\rho}$, σ_1, $\bar{V}$, and a and b coefficient were evaluated. The correlation $\bar{\rho}$ was changed about 50% (0.06 and 0.20, compared with 0.13). Because of the nonlinear way $\bar{\rho}$ enters into the standard deviation, equation 10, this leads to only about a 20% change in σ_n (1.02 and 1.71, compared with 1.44). The single site standard deviation σ_1, and the mean speed $\bar{V}_1$ were each changed 10%, as shown at the top of Table 9. The representative high and average variance a and b coefficients were also run and compared with the low variance reference case. Table 9 shows that the rms deviation between the observed and model array is about twice as sensitive to mean wind speed $\bar{V}_1$ as to the other parameters. A 10% change in $\bar{V}_1$ produces an increase in rms deviation from about 2% to about 5%, while a 10% change in standard deviation σ_1 or σ_n or power coefficient b value produces an increase in rms deviation from about 2% to about 3½%.

Thus, the most important parameter for careful selection of a "representative" site for the model input is the mean wind speed. All other parameters have significantly less importance. For cases in which the seasonal and/or diurnal variation of wind power is important, a "representative" site should be chosen which has appropriate seasonal and diurnal variations of its mean wind speed.

Table 8. Power Model Coefficients for Low, Average, and High Variance Reference Statistics*

Coefficient	Reference Statistics Variance low	average	high
α_0	-0.107	-0.074	+0.016
α_1	-0.190	-0.182	-0.161
α_2	-0.236	-0.219	-0.181
α_3	1.170	1.095	0.888
a	-0.44	-0.38	-0.25
b	1.17	1.10	0.89

* The following parameter values were used to evaluate the power model coefficients in this table:

$V_{in}/V_r = 5.3/10.6 = 0.5$

$V_{out}/V_r = 21.2/10.6 = 2.0$

$C_{pr}/C_{pm} = 1$

Table 9. Sensitivity Analysis of Array Model to its Various Input Parameters*

Power, MW	Reference (MKE) Model Array Power Distribution	Sensitivity of Array Power Distribution to: avg. correlation $\bar{\rho}$		standard deviation σ_1		Mean Speed $\bar{V}_1$		Power Parameters a, b	
		$\bar{\rho}=0.06$	$\bar{\rho}=0.20$	$\sigma_1=3.48$	$\sigma_1=2.84$	$\bar{V}_1=8.86$	$\bar{V}_1=7.25$	a=-0.38 b= 1.10	a=-0.25 b= 0.89
0	0.7	0.2	1.5	1.1	0.4	0.2	2.1	0.4	0.1
0-0.2	1.9	0.7	2.8	2.4	1.3	0.7	4.4	1.3	0.8
0.2-0.4	4.8	2.8	6.1	5.6	3.9	2.1	9.8	4.2	3.8
0.4-0.6	10.3	8.4	10.8	10.7	9.5	5.1	17.3	10.3	11.5
0.6-0.8	17.9	19.2	16.3	17.0	18.5	10.7	23.5	17.0	21.9
0.8-1.0	24.0	30.8	20.2	21.7	26.6	18.3	23.0	26.4	32.5
1.0-1.2	22.8	27.7	19.6	20.9	25.0	24.4	14.3	22.8	23.2
1.2-1.4	13.2	9.6	13.9	13.8	12.3	22.6	4.8	14.0	6.0
1.4-1.6	3.9	0.7	6.6	5.5	2.3	12.4	0.7	3.3	0.3
1.6-1.8	0.4	0.0	1.9	1.1	0.1	3.2	0.0	0.2	0.0
1.8-2	0.0	0.0	0.3	0.1	0.0	0.3	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
σ_n m/s	1.44	1.02	1.71	1.59	1.30	1.44	1.44	1.44	1.44
k_n	6.46	8.48	5.39	5.82	7.26	7.17	5.77	6.46	6.46
c_n m/s	8.63	8.53	8.74	8.69	8.60	9.46	7.82	8.63	8.63
rms deviation (%) (0-1.8 MW)	2.2	5.0	1.9	1.8	3.3	5.8	4.6	2.9	5.0

* Except as noted in this table, the following parameter values were used to evaluate distributions:

$\bar{V}_1$ = 8.05 m/s, σ_1 = 3.16 m/s, $\bar{\rho}$ = 0.13, a = -0.44, b = 1.17

Additional parameter values for σ_n, k_n, and c_n are given in the body of the table.

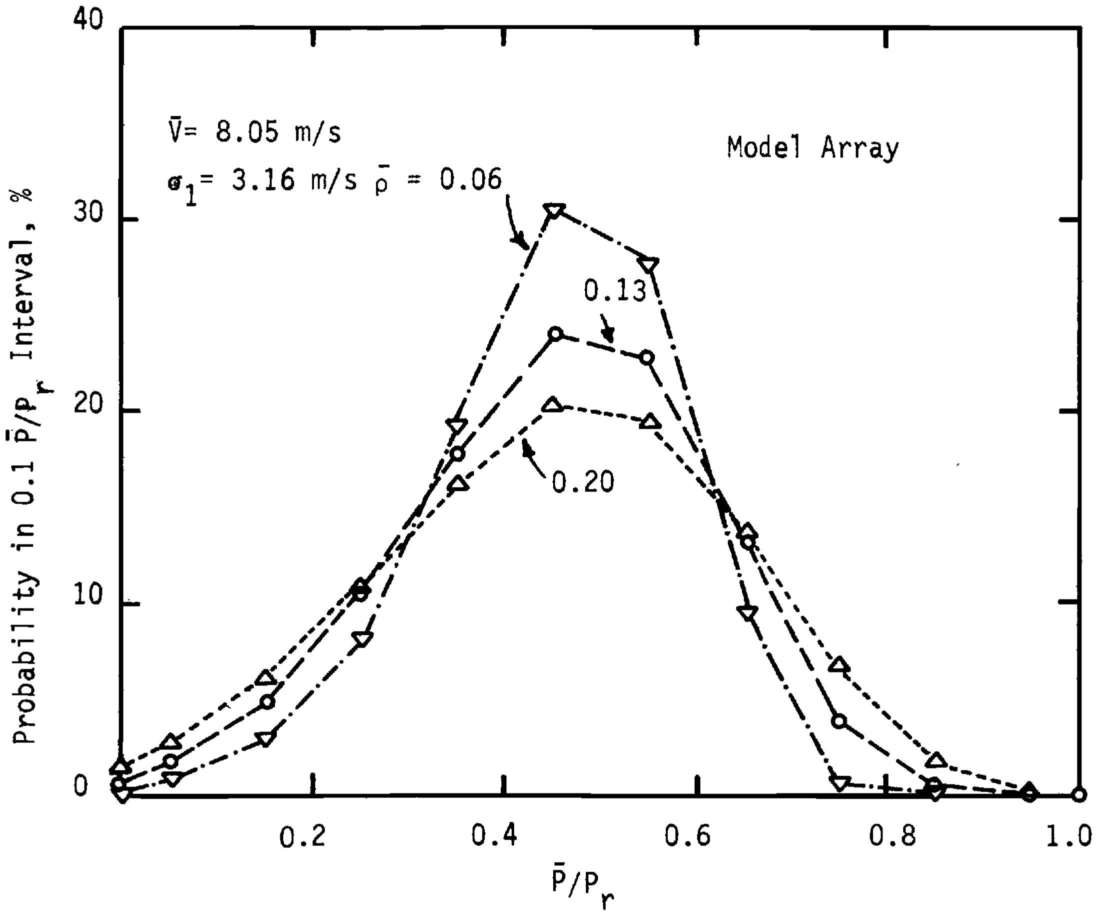

Figure 10. Sensitivity to Average Correlation $\bar{\rho}$

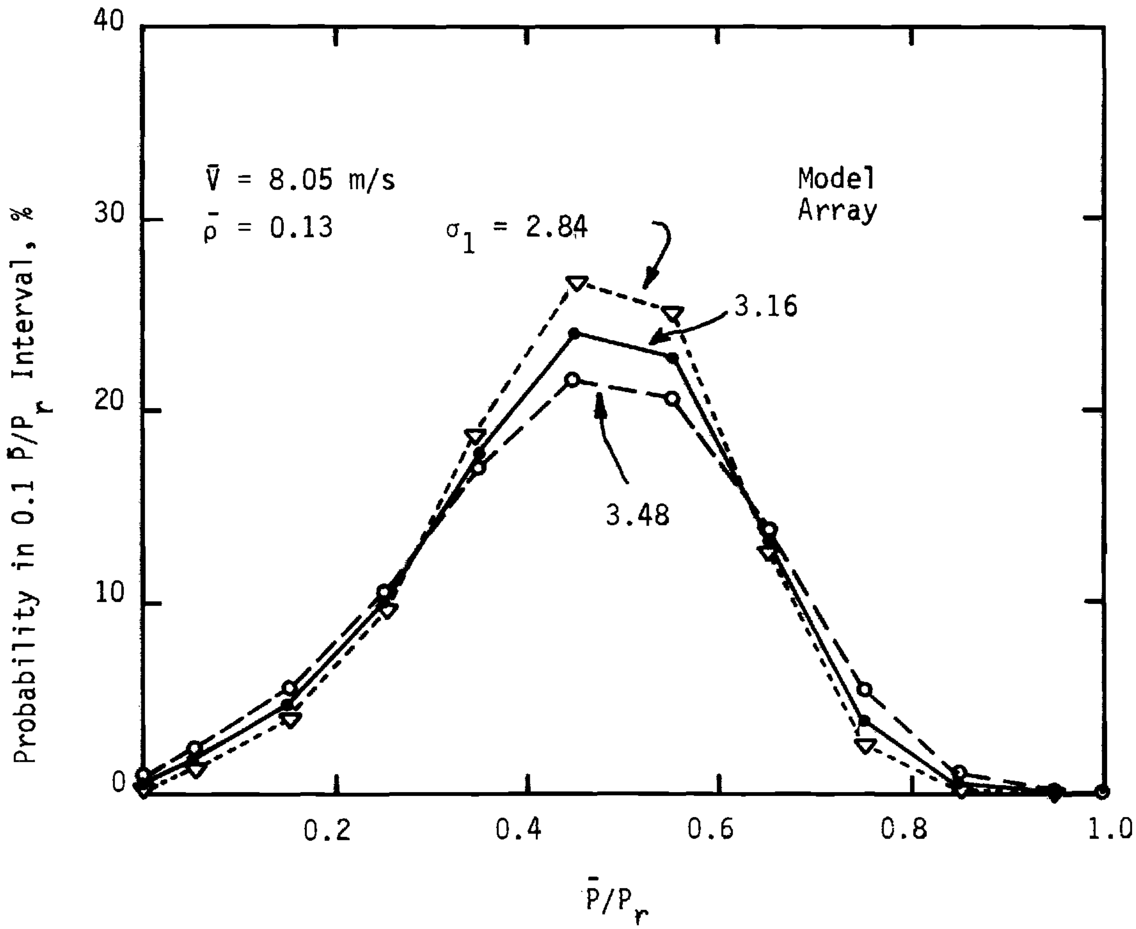

Figure 11. Sensitivity to Single Site Standard Deviation σ_1

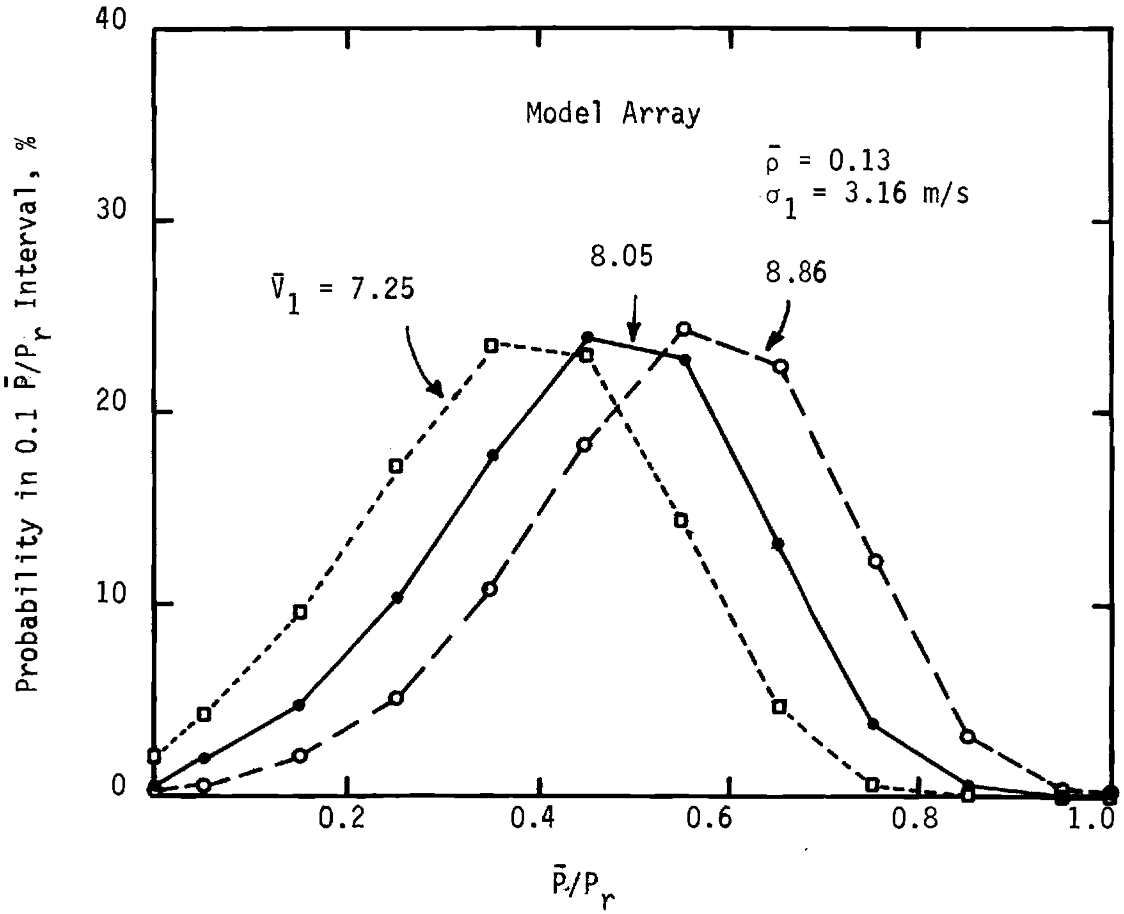

Figure 12. Sensitivity to Single Site Mean Wind $\bar{V}_1$

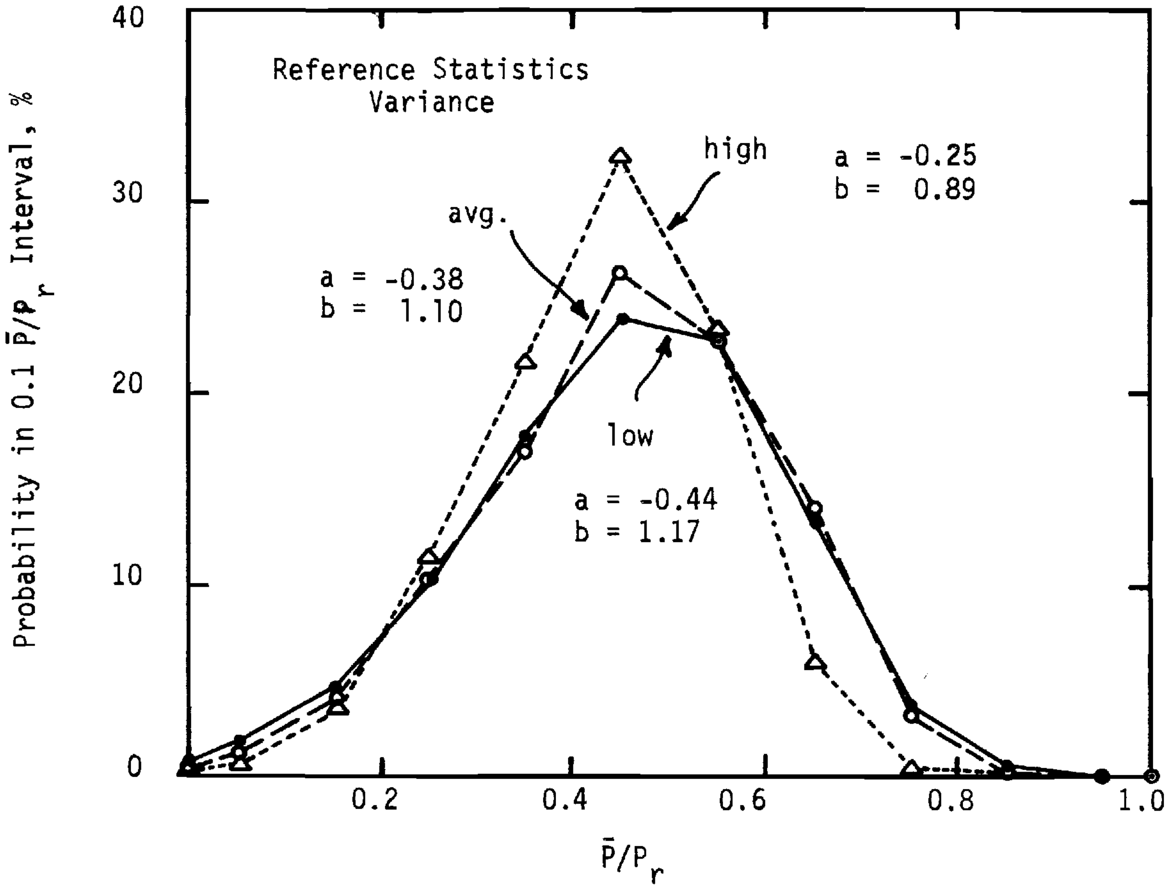

Figure 13. Sensitivity to Power Coefficients a and b for Low, Average, and High Reference Statistics Cases

4. EXAMPLE APPLICATION OF THE SIMPLIFIED ARRAY STATISTICS MODEL

This section develops and illustrates the process by which the distribution statistics of an array of WECS can be evaluated. The illustration is made with a particular example WECS design and array size. The steps of the methodology can be followed with equal ease of application for any other input parameter values, array sizes, or WECS operating characteristics.

Suppose that it is desired to evaluate the power distribution probabilities for an array of 200 kW WECS, whose parameters are given in Appendix D. Consider these WECS units to be spread over an array of maximum dimension R=300 km, in 20 autonomous WECS "farms" of 50 units each (1000 units in the array, total rated power 200 MW). Furthermore, suppose that the average 10 m level wind speed, either determined from averages at several meteorological sites within the array area, or from a single site taken to represent the average wind speed, is 6 m/s.

Step 1. Determine or estimate the mean wind speed at hub height for the array. In this case the mean wind $\bar{V}_{10}$ at 10 m level was presumed, or determined from available data. The equations to adjust this wind speed to hub height (30.5 m, 100 ft in this case) are those for projecting the Weibull c and k parameters (Justus, 1976). From Figure D-5, the average variance k_{10} value at 10 m for $\bar{V}_{10}$ = 6 m/s is k_{10} = 2.30. The Weibull c_{10} value at 10 m is $c_{10} = \bar{V}_{10}/\Gamma(1 + 1/k_{10}) = 6/\Gamma(1.43) = 6.77$. The Weibull scale factor $c_{30.5}$ for hub height is

$$c_{30.5} = c_{10}(30.5/10)^{\alpha} \tag{19}$$

where α is given by

$$\alpha = 0.37 - 0.088 \ln 6.77 = 0.20.$$

With this value for α, $c_{30.5}$ from equation 19 is 8.48. The Weibull shape factor for hub height is

$$k_{30.5} = k_{10}/[1 - 0.088 \ln (30.5/10)] = 2.55. \qquad (20)$$

These values for $c_{30.5}$ and $k_{30.5}$ can then be combined to yield the projected mean wind speed at hub height $\bar{V}_{30.5}$

$$\bar{V}_{30.5} = c_{30.5}\Gamma(1 + 1/k_{30.5}) = 8.48\Gamma(1.39)$$

$$= 7.5 \text{ m/s}. \qquad (21)$$

The above methodology assumes only the mean wind speed at 10 m level and uses the reference wind distribution statistics expected Weibull k value from Figure D-5. Another possibility is that the actual Weibull scale factor k_{10} may have been determined by analysis of wind data from one or more sites (in which case, this value could be used directly in equation 20 above. Alternatively, the standard deviation about the mean wind σ_{10} might be known at the 10 m level, in which case the equation (Justus and Hargraves, 1977a)

$$k_{10} = (\sigma_{10}/\bar{V}_{10})^{-1.086} \qquad (22)$$

may be used, with this k_{10} value then applied in equation 20.

Step 2. Determine or estimate the number of sites in the array. For this example, n was specified as 20. In the case where n is to be determined and the number of autonomous farm areas is not obvious (e.g. uniformly distributed WECS over a large area), the array region can be divided more-or-less arbitrarily into farm areas of about 20 km in dimension. Each of these "farm" areas can be considered a separate site for array analysis purposes. Each such site is assumed to have the same power output at a given time, with diversity only on the farm to farm basis. Earlier array correlation studies (Justus, 1976; Justus and Hargraves, 1977a) justify this approach. These studies showed a rapid drop in correlation occurring somewhere in approximately the first 25 km, from unity at zero separation (by definition) to about 0.8 at approximately 25 km. [That this effect is a real mesoscale phenomena and not anemometer error was justified by Justus (1976).]

Step 3. Determine the spatial cross correlation for the array. This is a two step process to estimate $\bar{\rho}$, unless cross correlative studies are done directly on simultaneous time series data from several sites in the study area. First the mean site separation $\bar{r}$ is calculated from the known maximum dimension R of the array (300 km in this example). From equation 2, $\bar{r}$ is calculated as follows:

$$\bar{r} = (n+1)R/[2.64(n-1)] = 21(300)/[2.64(19)] = 126 \text{ km.}$$

Next the correlation $\bar{\rho}$ appropriate to this mean separation is determined, e.g. from equation 4

$$\bar{\rho} = \exp[-(\bar{r}/520)^{0.57}] = \exp[-(126/520)^{0.57}] = 0.64.$$

Step 4. Determine the Weibull c_n and k_n parameters for the array distribution. From equation 13 adapted for use with the known $k_{30.5}$ instead of the $\sigma_0/\bar{V}_0$ ratio, the value of k_n is evaluated as

$$k_n = k_1 \; 0.893 \; \{[1 + (n-1)\bar{\rho}]/n\}^{-0.543} = 2.55 \times 0.893 \times \{[1 + 19(0.64)]/20\}^{-0.543} = 2.86,$$

where 0.893 is the empirical constant 1.11 to the -1.086 power. From the known $\bar{V}_n = \bar{V}_1 = 7.5$ m/s, c_n is determined by

$c_n = \bar{V}_n/\Gamma(1 + 1/k_n) = 7.5/\Gamma(1.35) = 8.43$ m/s.

Step 5. If distributions of array wind speed are desired, determine these with the c_n and k_n values found in step 4. From equation 15, the probability of finding V_{array} between any desired limits V_j and V_k is

$$p(V_j \leq V_{array} \leq V_k) = \exp[-(V_j/c_n)^{k_n}] - \exp[-(V_k/c_n)^{k_n}]. \qquad (23)$$

Table 10 gives example computed values using equation 23 and array c_n and k_n values (8.43 m/s, 2.86) compared with the single site speed distribution [computed using $c_{30.5} = 8.48$ m/s, $k_{30.5} = 2.55$].

Step 6. Compute array power output distribution. First, the array power model coefficients a and b must be evaluated. As given in Appendix D, a and b are determined from the linear regression coefficients, Table D-1 via equation 18, with hub height cut-in ratio V_{in}/V_r, hub height cutout ratio V_{out}/V_r, and power coefficient ratio C_{pr}/C_{pm}. For the 200 kW example WECS, with average variance α values, and $V_{out}/V_r = 2$,

$$\begin{aligned} a &= \alpha_0 + \alpha_1(V_{in}/V_r) + \alpha_2(C_{pr}/C_{pm}) \\ &= -0.035 - 0.182(4.2/10) - 0.219(0.66) = -0.26 \qquad (24) \\ b &= \alpha_3 = 1.03. \end{aligned}$$

These a and b values are then used to find the array speeds V_j and V_k which correspond to any desired power interval P_j to P_k. V_j and V_k are defined in equation 17 as

$$\begin{aligned} V_j &= [(P_j/P_r) - a]V_r/b \\ V_k &= [(P_k/P_r) - a]V_r/b. \end{aligned} \qquad (25)$$

Table 11 shows values of array speed and array power distribution, computed from equation 15.

Table 10. Calculated Example Array Speed Distribution Compared with Single Site Speed Distribution

Wind Speeds V_j, V_k, m/s	Calculated Single Site Speed Distribution, %	Calculated Array Speed Distribution, %
0-2	2.5	1.6
2-4	11.2	9.6
4-6	20.2	20.3
6-8	23.9	26.2
8-10	20.4	22.7
10-12	13.0	13.2
12-14	6.1	5.0
14-16	2.1	1.2
>16	0.6	0.2

Table 11. Calculated Example Array Power Distribution

Array Powers per WECS Unit P_j, P_k kW	Wind Speeds V_j, V_k, m/s	Calculated Array Power Probability, %	
0	$\leq$ 2.52	3.1	
0-50	2.52-4.95	16.5	91.4%
50-100	4.95-7.38	29.9	
100-150	7.38-9.81	29.1	
150-200	9.81-12.23	15.9	
200	$\geq$ 12.23	5.5	

$$p(P_j \leq P \leq P_k) = \exp\left[-(V_j/c_n)^{k_n}\right] - \exp\left[-(V_k/c_n)^{k_n}\right] \tag{26}$$

with V_j and V_k from 25. Notice that the effective cut-in speed, 2.52 m/s, is less than the cut-in for a single machine (4.2 m/s), and the effective rated speed for the array 12.23 m/s is above the single machine rated speed (10.0 m/s), for reasons explained in Section 3. For comparison with the array power distribution, the probability of single site power of zero, $p(V \leq 4.2$ m/s$)$, is 9.7%; probability of single site power equal to rated power, $p(V \geq 10$ m/s$)$, is 19.3%. This means that a single machine is operating between cut-in and rated 71% of the time, compared with the example array, which has power above zero and below rated 91.4% of the time.

5. CONCLUSIONS

Array spatial correlation results, summarized from the earlier array studies in four regions, plus the multiregional array results of Appendices A to C, indicate that average array correlation ($\bar{\rho}$) is a function of the average separation ($\bar{r}$), as given by equation

$$\bar{\rho}(\bar{r}) = \exp[-(\bar{r}/520)^{0.57}], \tag{27}$$

where the average separation ($\bar{r}$) can be evaluated from maximum array site pair separation ($\bar{R}$) and number of array sites (n) by equation 2

$$\bar{r} = (n+1)R/[2.64(n-1)]. \tag{28}$$

The correlation results in the Pacific Coast region, probably because of significant terrain influence, were approximately 45% lower than given by equation 7. Hence, substantially lower correlations than those given by this equation might be more suitable for arrays in irregular terrain regions. For a given terrain situation, it would be best, if possible, to do actual cross-correlations between sites and average these, using equation 8, rather than relying on equations 27 and 28.

The simple array statistics model, developed in the previous report (Justus and Hargraves, 1977a), was applied in general cases with both array average statistical input and single "representative" site input. Either approach yields adequate simulation capability for the array wind speed and power distributions. However, if seasonal and diurnal speed and power variations are an important factor, the "representative" site should be selected not only on the basis of annual mean speed but also on suitability of its seasonal and diurnal variations of mean wind speed.

With the relations 27 and 28 above for determining spatial correlation and with mean wind and number of array sites specified, the only additional input required for the model array speed and power distribution is information on the rms average standard deviation over the individual sites of the array (or equivalently the appropriate Weibull shape factor k). These data may be estimated from reference distribution statistics (Figure D-5) if measurements of σ or k are not available.

The multiregional array statistics observed and reported in Appendices A to C provide valid testing examples for the array model (Figures C-1 through C-12). Beyond about 1500 km no further decrease occurs in spatial correlation (see Figure B-4); therefore, an upper limit exists to the effective size of arrays for marginal benefits from the array diversity effects. Thus, the regional-size arrays, studied earlier, probably offer the maximum benefits from the array smoothing and diversity for increasing wind power availability and producing capacity credit.

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APPENDIX A

WIND STATISTICS OF THE MULTIREGIONAL ARRAYS

This Appendix documents the wind speed statistics of the selected sites and arrays of the multiregional and continental array analysis. Site names and identification codes are given in Table 1 and site locations are shown in Figure 1 in Chapter 1.

Sites for the multiregional array were selected as the higher annual mean wind speed sites from previously reported array studies in New England and Central U. S. (Justus, 1976) and in the Great Lakes and Pacific Coast (Justus and Hargraves, 1977a). Since different years were used in these earlier studies, a common time period of analysis, 1971-1973, was selected for the multiregional analysis.

Table A-1 shows seasonal hub height (60 m) mean wind speeds for each of the multiregional array sites. The seasons are defined as: winter, days 1-44 and 315-366; spring, days 45-134; summer, days 135-224; and fall, days 225-314. Wind speed data are from three-hourly National Weather Service data (one minute averages on the hour), converted from anemometer level (Z_a) to hub height (Z) by the speed dependent power law relation:

$$V(Z) = V_a(Z/Z_a)^{\alpha} \quad \text{(A-1)}$$

where V_a is the observed wind speed at anemometer height and the exponent α is given by

$$\alpha = (0.37 - 0.088 \ln V_a)/(1 - 0.088 \ln Z_a/10) \quad \text{(A-2)}$$

(Justus and Mikhail, 1976). Although this height projection method does not have the complete rigor of the Monin-Obukov scaling law (Panofsky,

Table A-1. Mean Wind Speed (m/s) at 60 m (197 ft) Hub Height for Continental Array Sites

Site Code	Winter	Spring	Summer	Fall	Annual
ACY	7.9	8.3	6.5	6.7	7.3
AMA	9.1	10.4	8.9	9.1	9.4
BLU	12.4	11.5	9.7	10.6	11.1
BOS	9.1	8.0	6.5	7.9	7.9
BUF	9.4	8.2	7.2	7.4	8.1
CNK	8.5	9.1	7.9	8.1	8.4
DDC	8.8	9.7	8.3	8.4	8.8
DTW	8.3	7.9	6.4	7.0	7.4
ERI	9.1	8.1	6.8	7.6	7.9
GLD	8.9	9.8	8.6	8.7	9.0
GRI	8.2	9.1	7.4	7.8	8.2
LGA	9.1	8.9	7.3	7.9	8.3
MFD	9.1	8.5	6.6	7.4	7.9
MKE	9.0	8.5	7.5	7.9	8.2
MKG	8.9	8.0	6.5	7.5	7.7
OKC	8.8	9.5	7.5	8.0	8.5
OTH	7.0	7.5	7.7	6.7	7.2
PVD	8.0	8.0	6.6	6.8	7.3
RSL	8.3	9.0	8.3	7.8	8.4
SDB	8.9	9.2	7.3	7.9	8.2
SFO	5.7	8.0	8.4	6.7	7.2
SMP	8.1	7.9	7.6	7.5	7.8
	---	---	---	---	---
AVG.	8.7	8.8	7.5	7.8	8.2

1977; Justus, 1978), it is somewhat realistic in that low nighttime winds will scale with higher power law than higher daytime winds. Thus, nighttime winds at hub-height may scale to values comparable to those for daytime winds at hub height. This means that the diurnal cycle of winds at hub-height is not forced to be the same magnitude or phase as the diurnal cycle of winds at anemometer height.

Seasonal and diurnal variations of mean wind speed at hub height are given in Figures A-1 and A-2 and Table A-2 for the Northeast and Great Lakes array: in Figures A-3 and A-4 and Table A-3 for the Great Lakes and Central array, in Figures A-5 and A-6 and Table A-4 for the Central and Pacific array, and in Figures A-7 and A-8 and Table A-5 for the Continental array. Comparison of Figures A-2, A-4, and A-6 to comparable data for the Great Lakes array and Pacific array (Justus and Hargraves, 1977a) indicates that the diurnal and seasonal pattern for the Great Lakes and Central array is similar to that for the Great Lakes array alone, while the diurnal and seasonal patterns for the Central and Pacific array are similar to that found for the Pacific array alone. (The diurnal variations for the Northeast and the Central arrays were not evaluated in the earlier studies.) For the Central and Pacific array the strongest average winds are in the summer afternoon, while winter winds in the morning hours are stronger than the summer winds in the morning (characteristic of the Pacific array sites).

Seasonal and diurnal variations of mean wind speed at hub-height for the Continental array are given in Figures A-7 and A-8 and Table A-5. Figure A-7 shows the Continental array seasonal variation to be closer to that of the Great Lakes and Central array than to the other arrays. Diurnal variations in the Continental array (Figure A-8) are

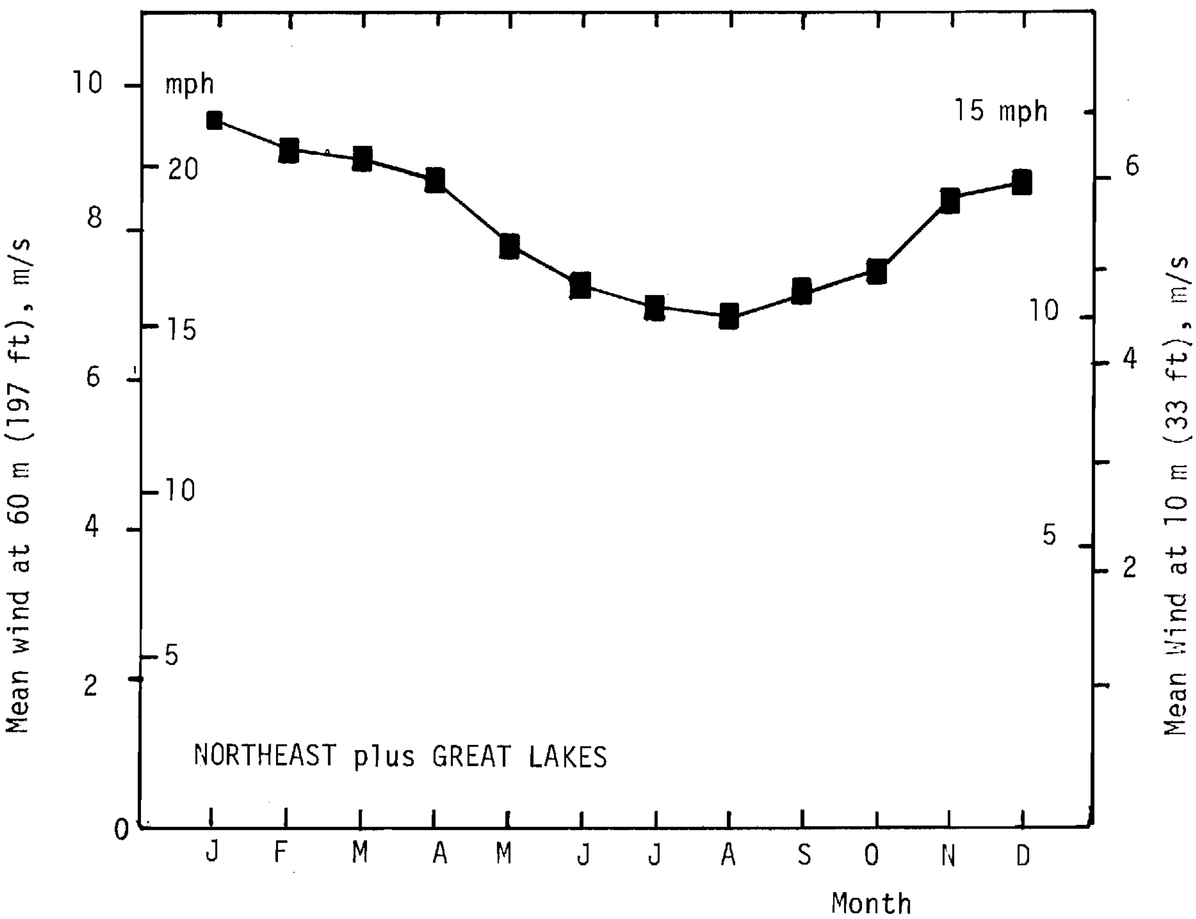

Figure A-1. Mean Winds Annual Variation for the Northeast plus Great Lakes Region.

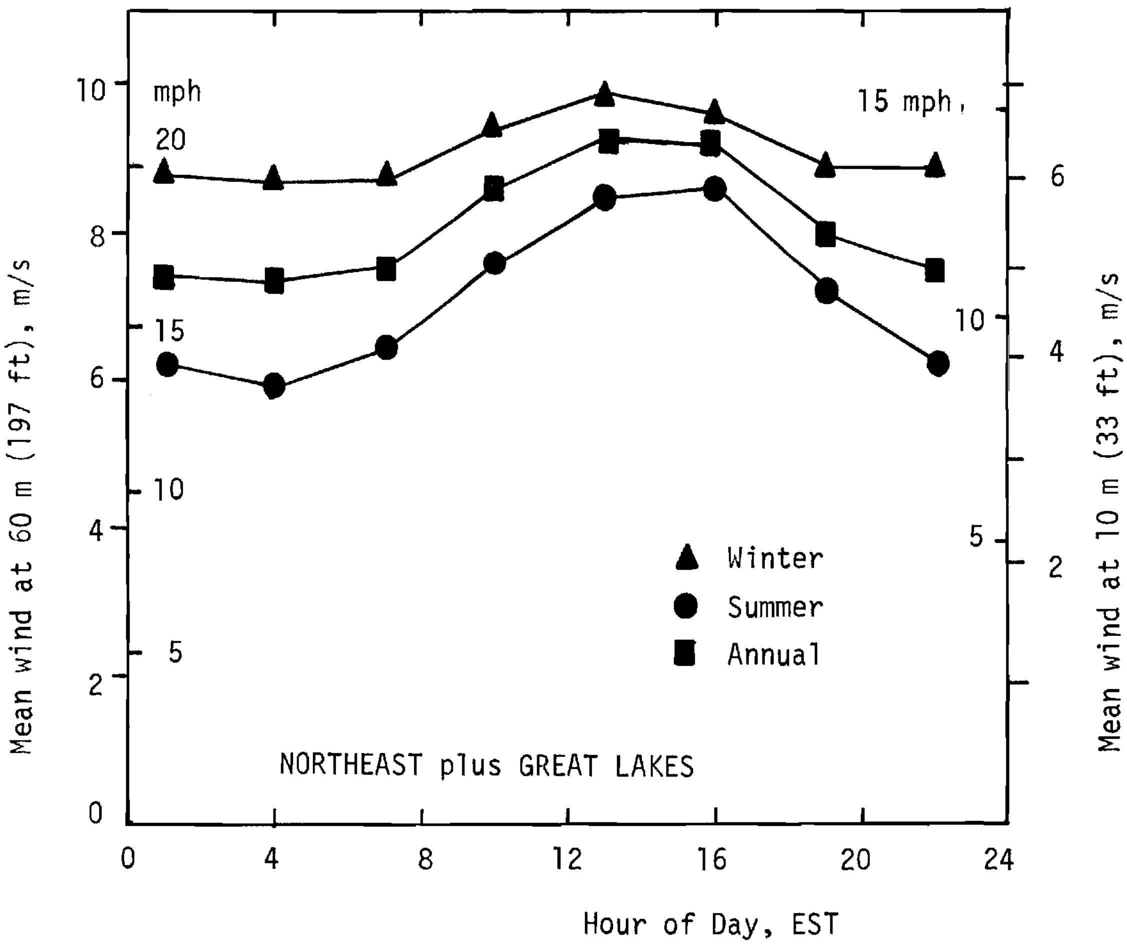

Figure A-2. Diurnal Variation of Mean Wind by Season for the Northeast plus Great Lakes Region.

Table A-2. Seasonal and Diurnal Variations of 60 m (197 ft) Hub Height Mean Wind Speed (m/s) for Northeast and Great Lakes Array

Season	Hour 1	4	7	10	13	16	19	22	Avg.
Winter	8.8	8.7	8.7	9.4	9.9	9.6	8.9	8.9	9.1
Spring	7.6	7.5	7.9	9.1	9.8	9.9	8.6	7.7	8.5
Summer	6.2	5.9	6.4	7.6	8.5	8.6	7.2	6.2	7.1
Autumn	7.2	7.0	7.1	8.3	8.8	8.6	7.4	7.3	7.7
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Annual	7.4	7.3	7.5	8.6	9.3	9.2	8.0	7.5	8.1

Wind speed, m/s

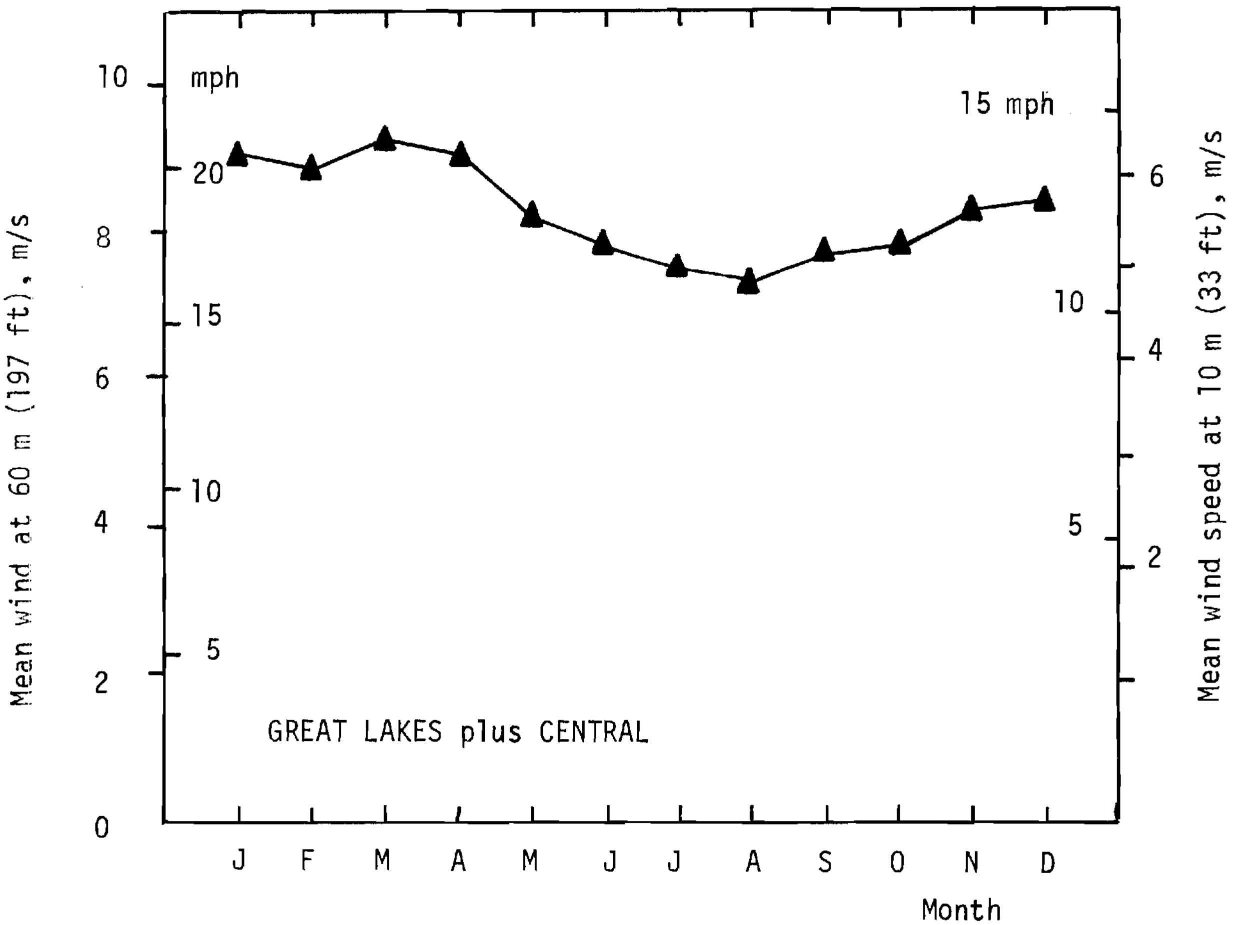

Figure A-3. Mean Winds Annual Variation for the Great Lakes plus Central Region.

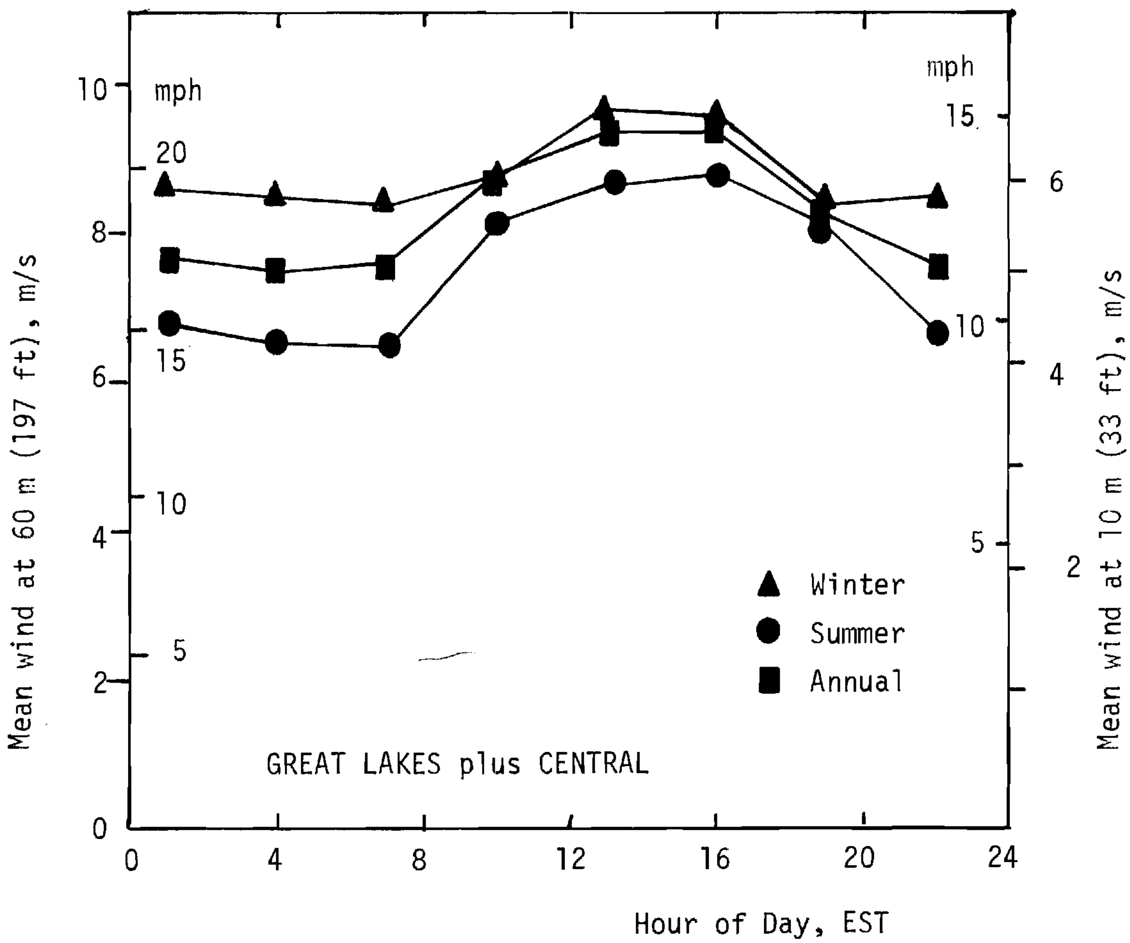

Figure A-4. Diurnal Variation of Mean Wind Speed by Season for the Great Lakes plus Central Region.

Table A-3. Seasonal and Diurnal Variations of 60 m (197 ft) Hub Height Mean Wind Speed (m/s) for Great Lakes and Central Array

Season	Hour 1	4	7	10	13	16	19	22	Avg.
Winter	8.6	8.5	8.4	8.8	9.7	9.6	8.4	8.5	8.8
Spring	8.1	8.0	8.0	9.6	10.2	10.2	9.2	7.9	8.9
Summer	6.8	6.5	6.5	8.2	8.7	8.8	8.1	6.7	7.5
Fall	7.4	7.3	7.3	8.4	9.1	9.0	7.5	7.4	7.9
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Annual	7.7	7.5	7.6	8.8	9.4	9.4	8.3	7.6	8.3

Wind speed, m/s

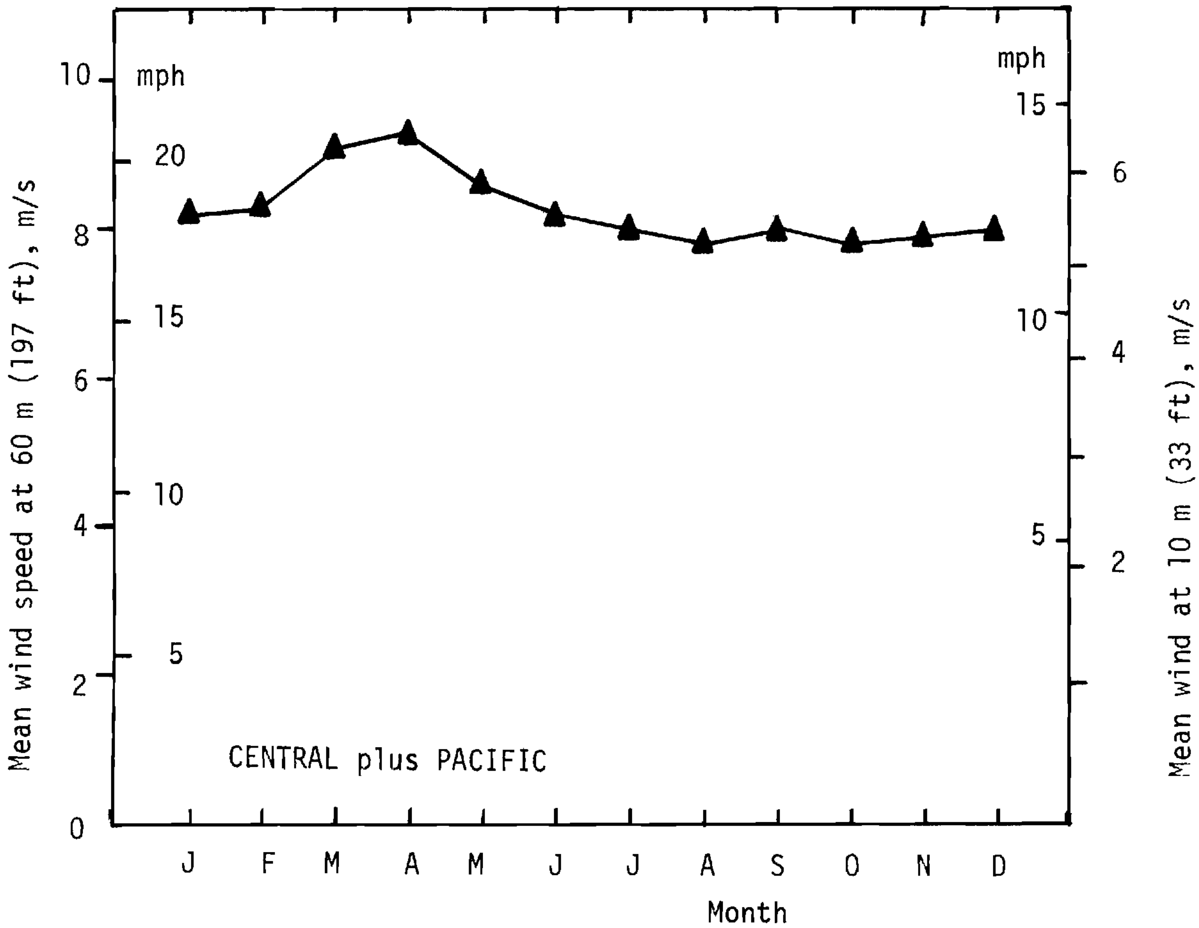

Figure A-5. Mean Winds Annual Variation for the Central plus Pacific Regions.

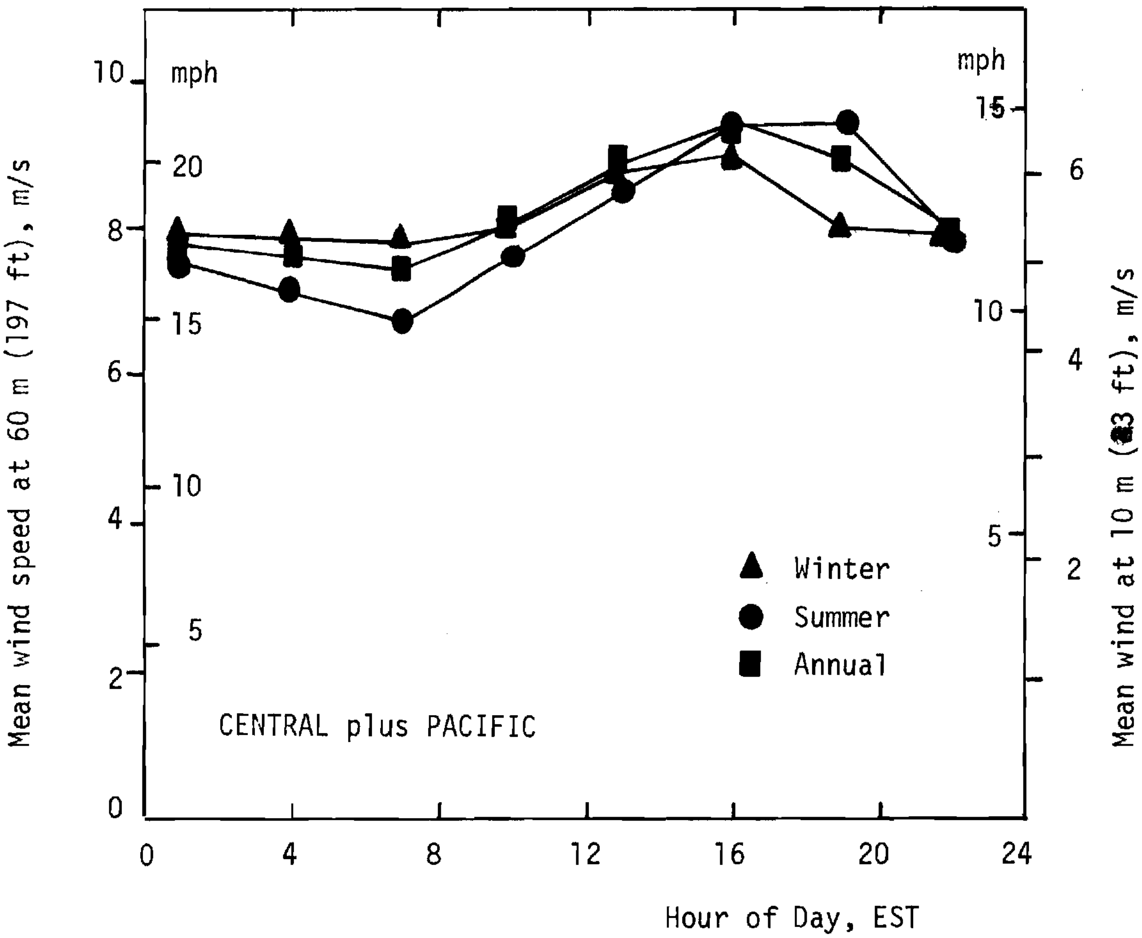

Figure A-6. Diurnal Variation of Mean Wind Speed by Season for the Central and Pacific Region.

Table A-4. Seasonal and Diurnal Variations of 60 m (197 ft) Hub Height Mean Wind Speed (m/s) for Central and Pacific Array

Wind speed, m/s

Season	Hour 1	4	7	10	13	16	19	22	Avg.
Winter	7.9	7.9	7.8	8.0	8.8	9.0	8.0	7.9	8.2
Spring	8.4	8.2	8.0	9.0	9.7	10.2	9.9	8.6	9.0
Summer	7.5	7.1	6.7	7.6	8.5	9.4	9.4	7.9	8.0
Fall	7.4	7.2	7.2	7.8	8.6	9.0	8.2	7.6	7.9
-----	---	---	---	---	---	---	---	---	---
Annual	7.8	7.6	7.4	8.1	8.9	9.4	8.9	8.0	8.3

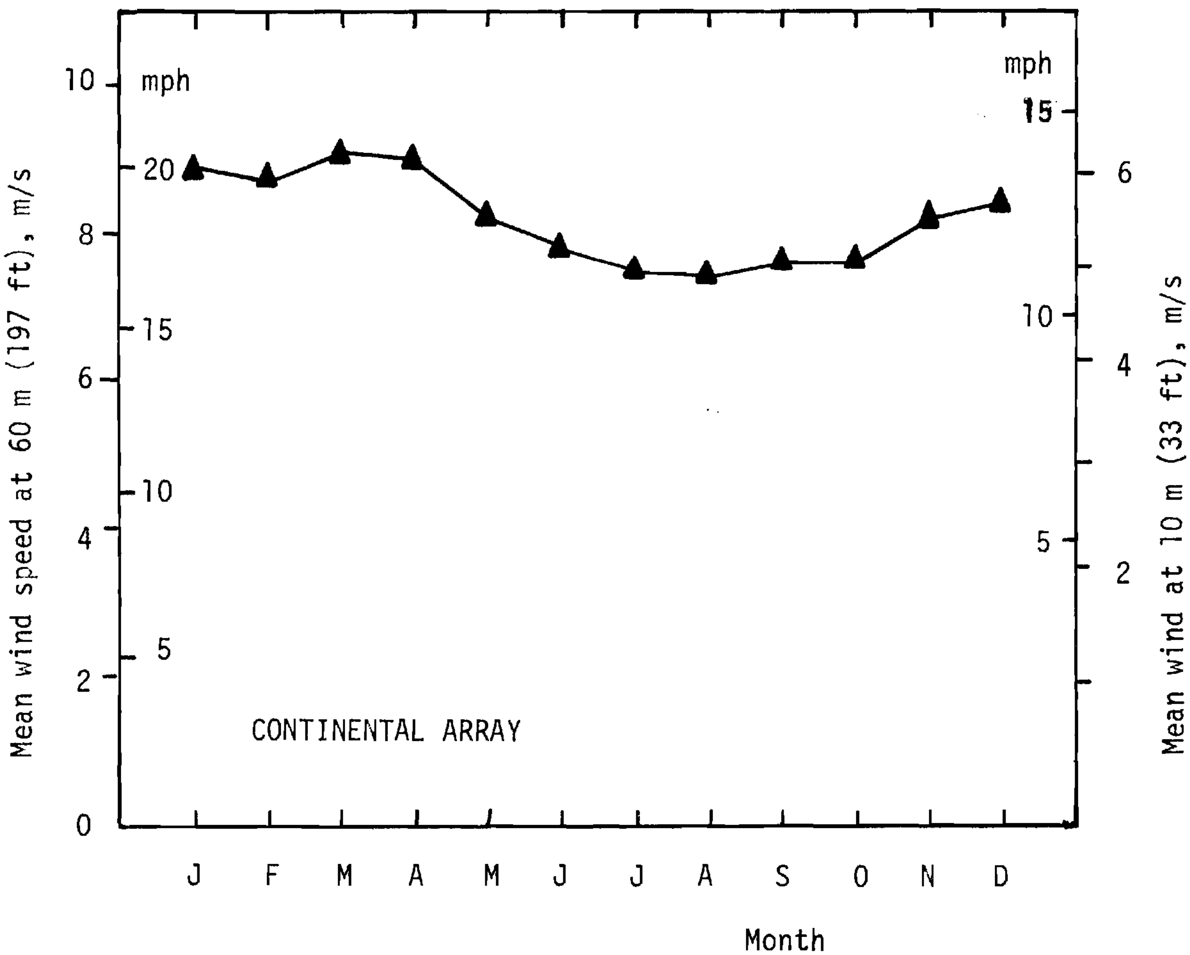

Figure A-7. Mean Winds Annual Variation for the Continental Array.

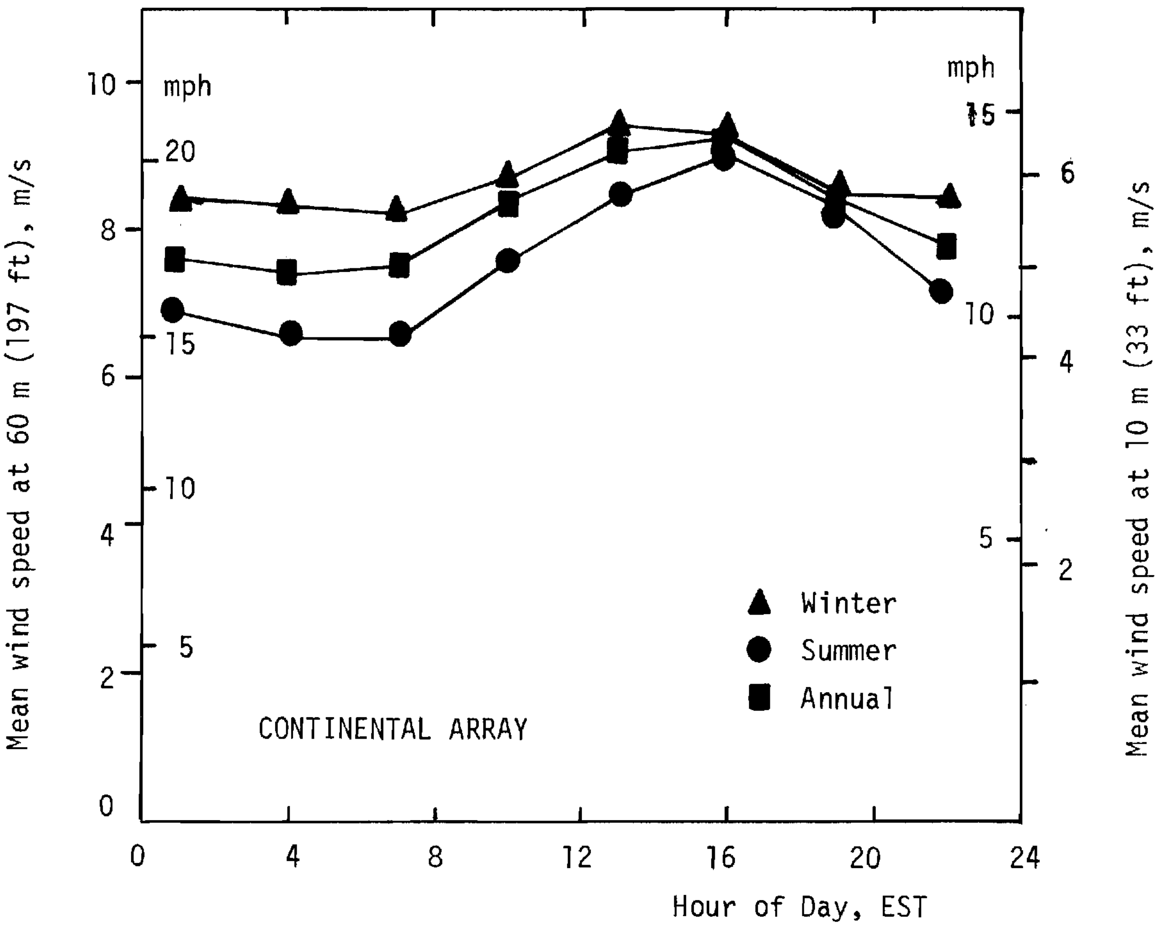

Figure A-8. Diurnal Variation of Mean Wind Speed by Season for the Continental Region.

Table A-5. Seasonal and Diurnal Variations of 60 m (197 ft) Hub Height Mean Wind Speed (m/s) for Continental Array

Season	Hour								
	1	4	7	10	13	16	19	22	Avg.
Winter	8.4	8.3	8.2	8.7	9.4	9.3	8.5	8.4	8.7
Spring	8.0	7.8	7.9	9.1	9.8	10.1	9.3	8.2	8.8
Summer	6.9	6.5	6.5	7.6	8.5	9.0	8.3	7.1	7.5
Fall	7.3	7.1	7.1	8.1	8.7	8.8	7.8	7.5	7.8
	---	---	---	---	---	---	---	---	---
Annual	7.6	7.4	7.5	8.4	9.1	9.3	8.4	7.8	8.2

Wind speed, m/s

smoothed by the combination of two effects: 1) the phase differences in diurnal cycle in the different time zones (common timing in Eastern Standard Time is used, rather than local standard or solar time), and 2) the summer and winter diurnal cycle is reversed in the Pacific array, as noted above.

APPENDIX B

TEMPORAL AND SPATIAL CORRELATIONS

Time Autocorrelation

Previous studies of time autocorrelation in the New England and Central United States (Justus, 1976) and Great Lakes and Pacific Coast (Justus and Hargraves, 1977a) have found a common pattern of strong diurnal influence in summer, with little or no diurnal influence on winter correlations. These features are exemplified by the Continental array average time autocorrelations shown in Figure B-1.

The time autocorrelation function for a given site is the normalized mean cross-product of departures of one-minute average wind speeds from the monthly mean wind speed for the site. The results of Figure B-1 are averaged over all of the sites of the Continental array (see Table 1) and over the three study years, 1971-1973.

Qualitatively, the time autocorrelation remains large ($\rho \geq 0.5$) for the 3 hour time lag, indicating that three-hourly data adequately resolve the time structure of the wind data for statistical sampling purposes. This has been proved more quantitatively by Corotis (1977), who showed that only every 8th to 12th consecutive hourly reading (every 3rd or 4th 3-hourly reading) is statistically independent. However, for persistence (run duration) applications, Corotis has shown that a 3-hour data interval is not adequate.

Corotis (1977) also has analyzed the time autocorrelation function $\rho(t)$ in terms of an exponential and a diurnal harmonic, i.e.,

$$\rho(t) = (1-Z)e^{-at} + Z\cos(2\pi t/24). \qquad \text{(B-1)}$$

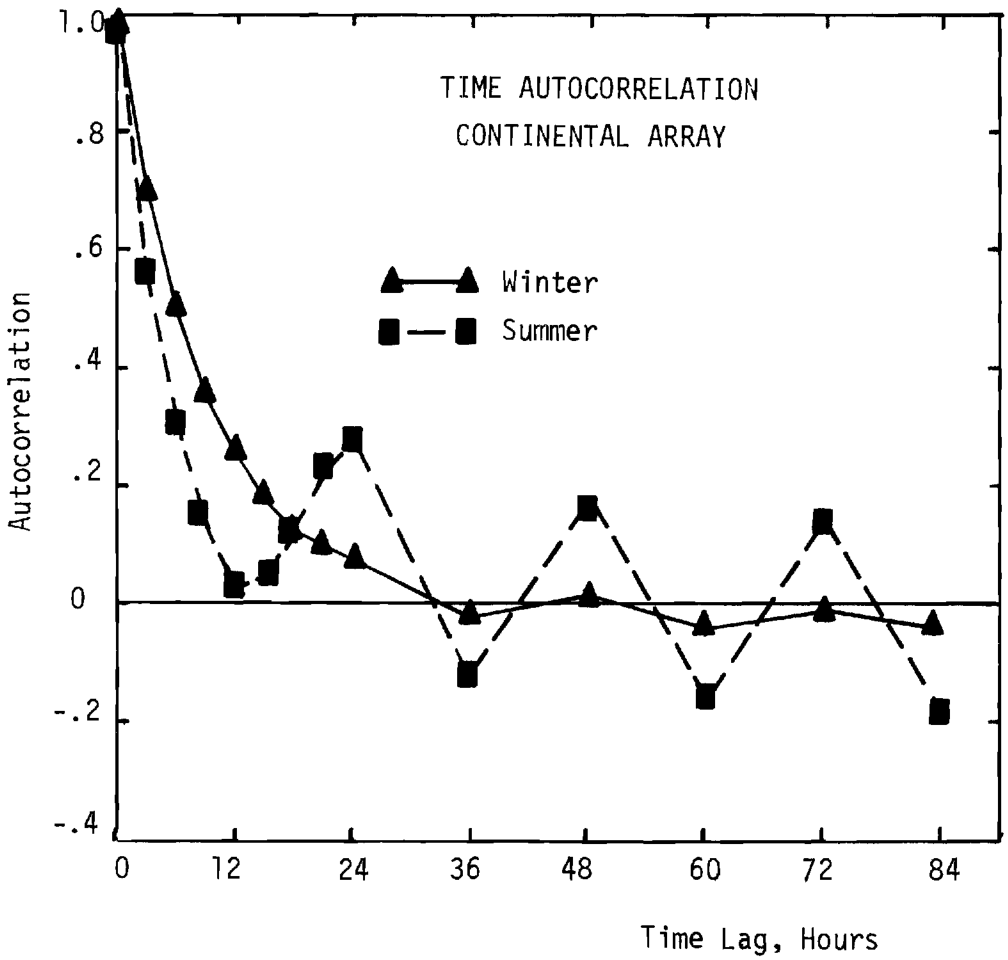

Figure B-1. Time Autocorrelation of Wind Speed for the Continental Array.

He found for Argonne, IL that the diurnal cycle was significantly less at the 45 m (150 ft) level than at the 5.5 m (18 ft) height. This is in agreement with findings of Crawford and Hudson (1973) on the WKY-TV tower in Oklahoma City. As pointed out in Appendix A, the speed dependent height projection method used here does not preclude a lessening of the diurnal cycle, since it uses a higher power law exponent for lower night time winds than for higher daytime winds. More rigorous Monin-Obukhov scaling may represent more adequately the true lessening of the diurnal cycle with height, but the full effect may not be realized in any height projection model until the loosely coupled vertical layering in the nocturnal boundary layer is appropriately simulated.

Spatial Cross-Correlation

The positive benefits of spatial diversity for arrays are characterized by the average spatial cross-correlation for the array. Variations of spatial cross-correlation with distance within various arrays and of average array spatial cross correlation versus average array separation distance have been studied (Justus, 1976; Justus and Hargraves, 1977a) and are summarized in the main body of this report.

The spatial cross-correlation for a given pair of sites is defined as the normalized mean cross-product of their (simultaneous) departures of one-minute mean wind speed from their respective monthly mean wind speeds for the site. For summary purposes, spatial cross-correlations have been averaged by intervals of intersite separation value (e.g. all 0-50 km separation data averaged, all 50-100 km, all 100-150 km, etc.) Figure B-2 shows the 1971-1973 average winter and summer spatial cross-correlation versus separation for the Continental array. These curves are very similar to corresponding ones for the respective individual arrays, except for the Pacific Coast array, which showed significantly lower spatial correlations in the 50-550 km separation range ($\rho \approx 0.3$ summer, 0.4 winter at 75 km separation in the Pacific Coast region).

If all of the independent site pairs i and j (numbering N) in an array have the set of spatial cross correlations ρ_{ij} versus their inter-site separation distances r_{ij}, then the array average spatial cross-correlation $\bar{\rho}$ is

$$\bar{\rho} = \sum_i \sum_j \rho_{ij}/N \tag{B-2}$$

and the average spatial separation $\bar{r}$ is

$$\bar{r} = \sum_i \sum_j r_{ij}/N \tag{B-3}$$

Figure B-3 shows a plot of $\bar{\rho}$ versus month for the Continental and two of the multiregion arrays. These data exhibit much less seasonal variation than did the average spatial cross-correlation data for the individual arrays, partially because of different seasonal phases of this parameter in the individual arrays ($\bar{\rho}$ had a winter peak in New England and the Great Lakes, a summer peak for the Pacific Coast array, and a spring/fall peak for the Central U. S. array).

Before the study of the multiregional arrays was begun, it was hypothesized that the alternating high and low pressure synoptic weather regimes at any given time across the Continental United States would lead to negative spatial cross-correlations at separations greater than about 1000-1500 km. This conjecture also seemed justified on the basis of extrapolations from the individual regional array analyses already done for separation distances up to 800-1000 km. If such were the case,

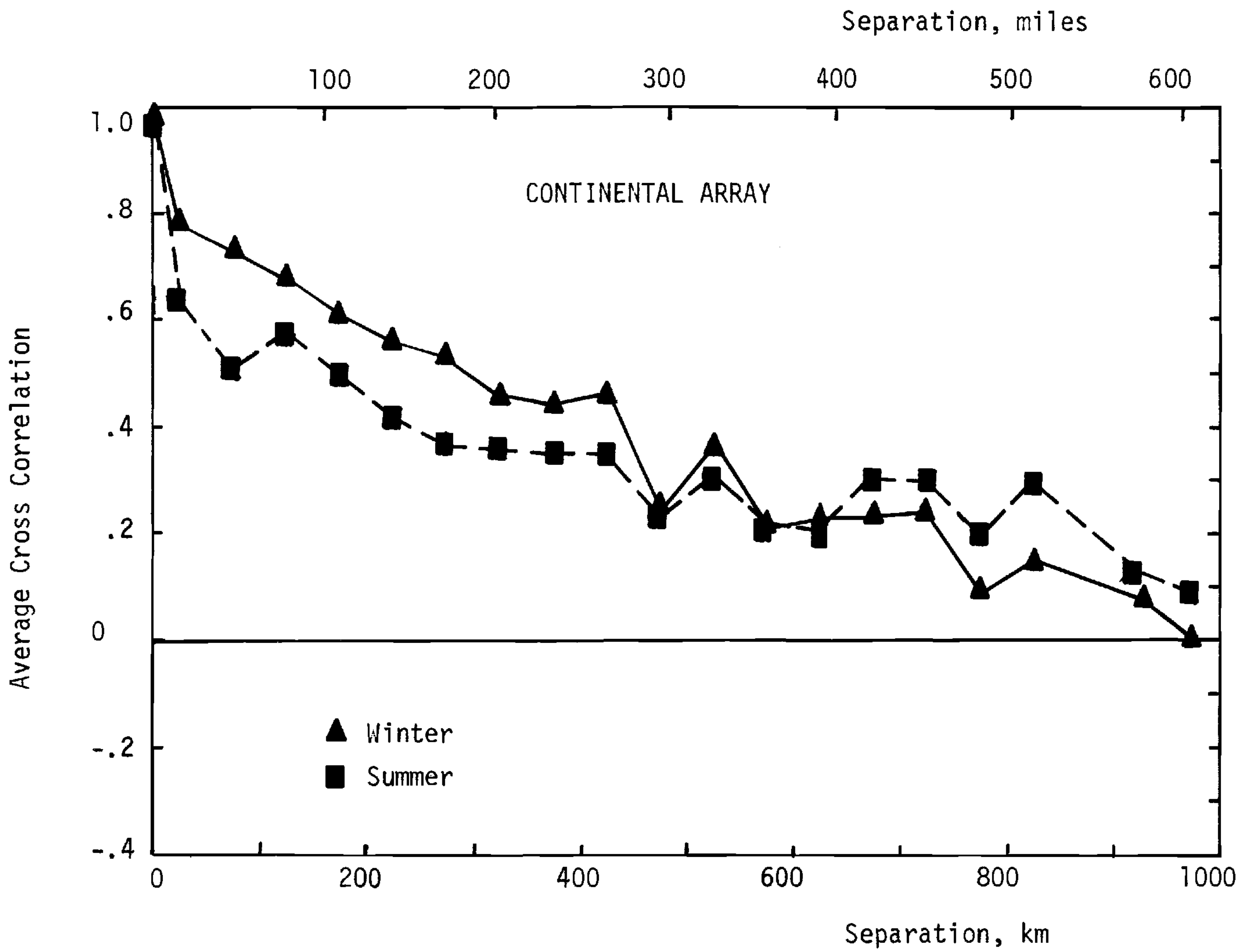

Figure B-2. Average Spatial Cross-Correlation Versus Separation for the Continental Array.

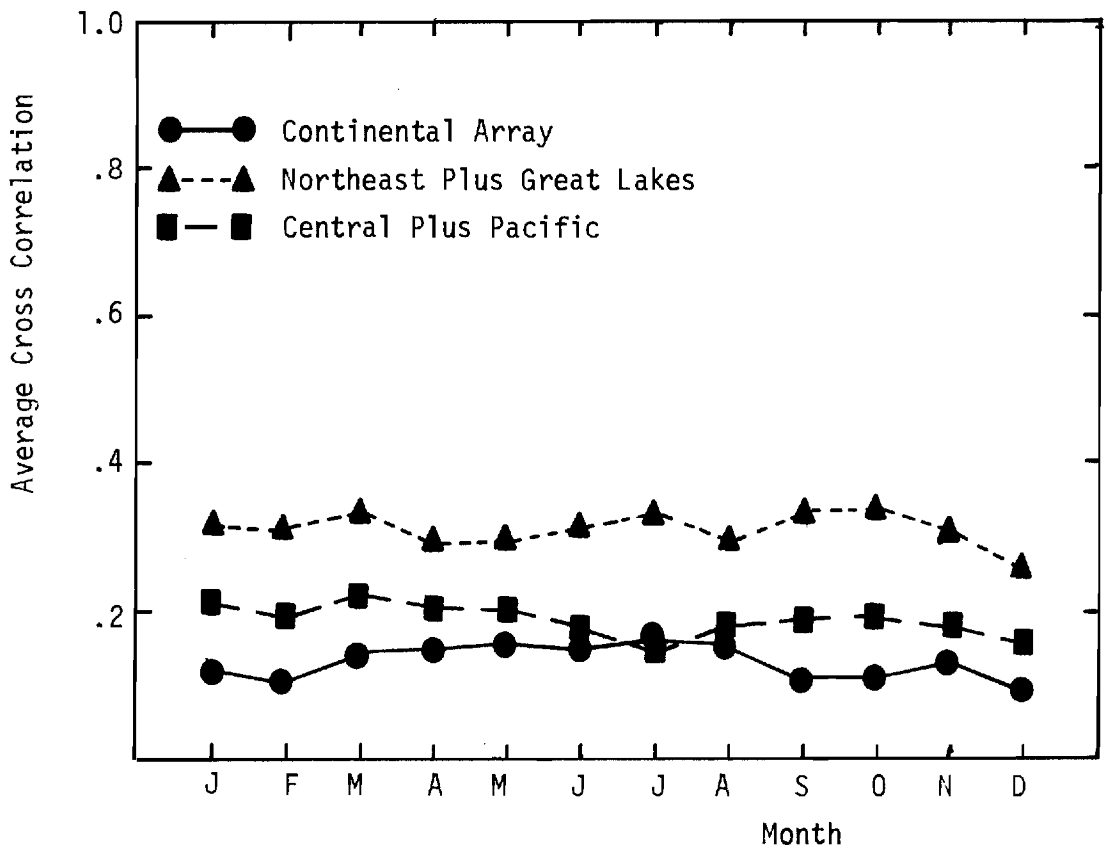

Figure B-3. Average Spatial Cross-Correlation Versus Month.

then significant additional array diversity effects might be obtained from arrays exceeding 1000 km in size. Unfortunately, this hypothesis turned out to be incorrect, as shown by the observed spatial cross-correlations for the Continental array plotted in Figures B-2 and B-4. Figure B-4 shows the 1971 average spatial cross-correlations versus separation for the Continental array, including all intersite separations. In Figure B-2, which plots 1971-1973 average data, the data points plotted at 975 km are the average for <u>all</u> site separations greater than 950 km. These data, together with the array average spatial correlation data of Figure B-3, show a "saturation" effect, in the terminology of Kahn (1978), because of a decline in marginal benefit of arrays toward decreasing the cross-correlation, i.e., increasing the wind diversity effect beyond 1000 km separation.

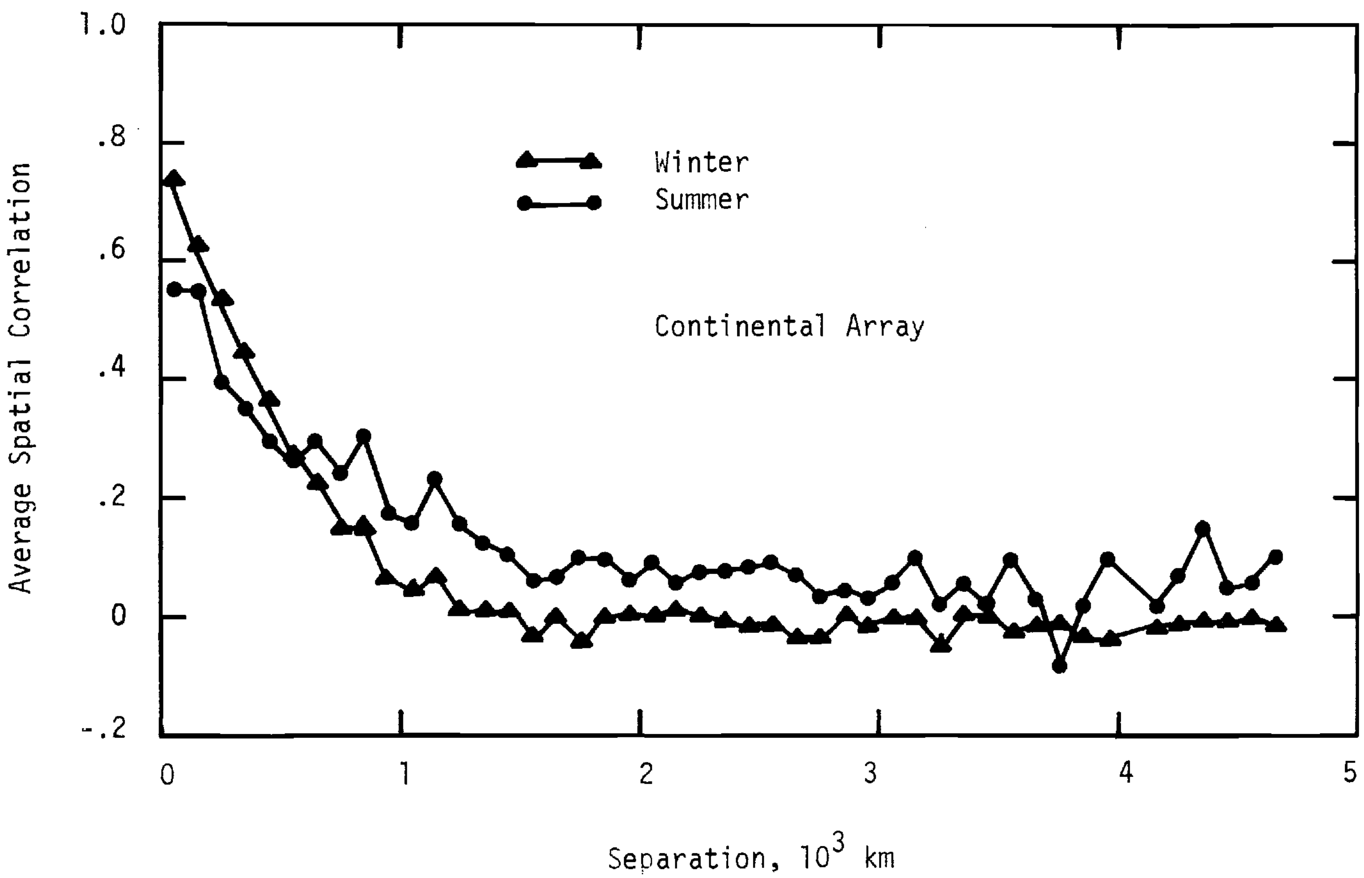

Figure B-4. 1971 Spatial Cross-Correlation for the Continental Array Including Data from All Inter-Site Separations.

APPENDIX C

WIND POWER OUTPUT STATISTICS

One minute average wind speeds, adjusted to hub height by the method described in Appendix A, were used in a generic power output curve model to simulate power output from a hypothetical 2MW rated power wind turbine. The power output curve model, described fully in Justus and Hargraves (1977a) starts at zero power at hub height cut-in wind speed and rises, by a generic parabolic curve, to full rated power at hub height rated speed. Between rated speed and the high-level cut-out speed, the power output curve remains at the rated output power. For the 2 MW wind turbine simulated here, the hub height cut-in speed was taken as 5.3 m/s (11.9 mph), and the hub height rated speed as 10.6 m/s (23.7 mph), with rotor diameter of 91.4 m (300 ft) and hub height of 60 m (197 ft).

Computed output power at each array site for each 3 hourly wind speed observation was summed across the array for "observed" array output. Frequency distributions of array output power were then evaluated by counting and "binning" these calculated array power values within power interval "bins". Frequency distributions of the individual site power output values were similarly binned, and the average individual site power output distribution was evaluated by combining (averaging) all of these binned individual site distributions.

Model output array distributions can be computed from the following parameters: number of sites in the array, n; average spatial cross-correlation for the array, $\bar{\rho}$ (see Appendix B); mean hub-height wind speed for the array, $\bar{V}_n$; and rms standard deviation for the array of wind speed about the mean wind speed at the respective site. If the n sites have respective site mean speeds $\bar{V}_i$ and standard deviations σ_i, then array mean speed $\bar{V}_0$ is computed as

$$\bar{V}_0 = \sum_{i=1}^{n} \bar{V}_i/n \tag{C-1}$$

and array rms standard deviation σ_0 is evaluated by

$$\sigma_0 = [\sum_{i=1}^{n} \sigma_i^2/n]^{\frac{1}{2}}. \tag{C-2}$$

Equations C-1 and C-2 assume equal numbers of wind turbines at each site. If, on the other hand, there are variable numbers of machines m_i at the n sites, with total number of machines $M = m_i$, then $\bar{V}_0$ and σ_0 can be evaluated by weighted averaging

$$\bar{V}_0 = \sum_{i=1}^{n} m_i\bar{V}_i/M \tag{C-3}$$

$$\sigma_0 = [\sum_{i=1}^{n} m_i\sigma_i^2/M]^{\frac{1}{2}} \tag{C-4}$$

With $\bar{V}_0$ and σ_0 evaluated, and n and $\bar{\rho}$ known, then Weibull distribution parameters c_n and k_n for the array wind speed V_n can be evaluated by

$$k_n = (1.11\sigma_0/\bar{V}_0)^{-1.086}\{[1 + (n-1)\bar{\rho}]/n\}^{-0.543} \tag{C-5}$$

$$c_n = \bar{V}_0/\Gamma(1 + 1/k_n), \tag{C-6}$$

where Γ is the usual gamma function. Equation C-5 is a combination of equations B-2 and B-7 from Justus and Hargraves (1977a), with an empirical correction factor of 1.11 (see main text). Equation C-5 represents a generalization, to the case with non-zero correlation, of the relationship between array standard deviation, single-site standard deviation and number of sites in the array. The standard deviation of array-average wind speed σ_n, for an array with n sites, would be $\sigma_n = \sigma_i/n^{\frac{1}{2}}$ if all the single-site standard deviations (σ_i) were equal and the sites were uncorrelated. Equation C-5 also includes an empirical relationship, $k_n = (\sigma_n/\bar{V}_0)^{-1.086}$, which approximates adequately the theoretical relationship between Weibull shape parameter k_n and $\sigma_n/\bar{V}_0$. The empirical adjustment factor 1.11 is applied to the σ ratio (see Figure 5 and accompanying description in the main text).

The array capacity factor, or plant factor, which considers the whole array as a multi-unit generation plant, is the array total output power divided by the total rated power of the wind turbines in the array. This can also be expressed as the array power per unit $P(V_n)$ divided by P_r the rated power of the wind turbine design (assuming a common design at all installations). This array capacity factor $P(V_n)/P_r$ can be evaluated as a function of array average speed V_n (where $\bar{V}_n = \bar{V}_0$) by the linear regression relation (Justus and Hargraves, 1977b)

$$P(V_n)/P_r = a + b\ (V_n/V_r) \tag{C-7}$$

If the probability of array power being between selected levels P_j and P_k (e.g. P_j = 0.5 MW, P_k = 1 MW) is desired, then the Weibull cumulative distribution can be used, namely

$$p(P_j \leq P \leq P_k) = \exp[-(V_j/C_n)^{k_n}] - \exp[-(V_k/C_n)^{k_n}] \tag{C-8}$$

where c_n and k_n are from equations C-5 and C-6 and V_j and V_k are found, from inversion of equation C-7 by the relations

$$\begin{aligned} V_j &= [(P_j/P_r) - a]\ V_r/b \\ V_k &= [(P_k/P_r) - a]\ V_r/b \end{aligned} \tag{C-9}$$

For the multiregional and Continental arrays the values a = -0.44 b = 1.17 were used, appropriate average values of those found earlier from the individual regional arrays. From the linear regression studies of Justus and Hargraves (1977b), these values of a and b are consistent with the "low variance reference distributions." (For sensitivity estimates for a and b coefficient values, see discussion in the main text).

Calculated power distributions for the "observed" array and average individual site (see ¶2 of this Appendix) and model array distribution (from equations C-1 through C-9 are illustrated for the multiregional and Continental arrays in Figures C-1 through C-12 and Tables C-1 through C-8. Figures C-1 through C-4 and Tables C-1 and C-2 are for the Northeast and Great Lakes array, Tables C-3 and C-4 for the Great Lakes and Central array, Figures C-5 through C-8 and Tables C-5 and C-6 for the Central and Pacific array, and Figures C-9 through C-12 and Tables C-7 and C-8 for the Continental array. For each array the figures illustrate: the winter and summer array power distribution in cumulative form, $p(P \leq P_j)$ or $p(P \geq P_j)$ (Figures C-1, C-3, C-5, C-7, C-9, C-11) and in frequency distribution form, $p(P_j \leq P \leq P_n)$ (Figures C-2, C-4, C-6, C-8, C-10, C-12). Note that in all of these cases the model array calculations seem to reproduce adequately the shape and details of the observed array distributions. The tables give, by season and by hour, the availability values $p(P \geq P_j)$ either for 200 kW per 2 MW generator (10% capacity factor) or for 500 kW per 2 MW generator (25% capacity factor). These power availability figures would apply as the same capacity factor values to any rated power machine, provided the cut-in and rated speeds are unchanged. For machines of different rated speed, the availability values would be different because of the dependence of equation C-9 on V_r.

Notice the number of seasonal and hourly values for which 25% capacity factor power availability exceeds 90% for the Continental and each of the multiregional arrays. This availability level would allow significant capacity credit to be assigned to these wind power arrays. For computational methods for capacity credit evaluation see JBF (1978) and Kahn (1978).

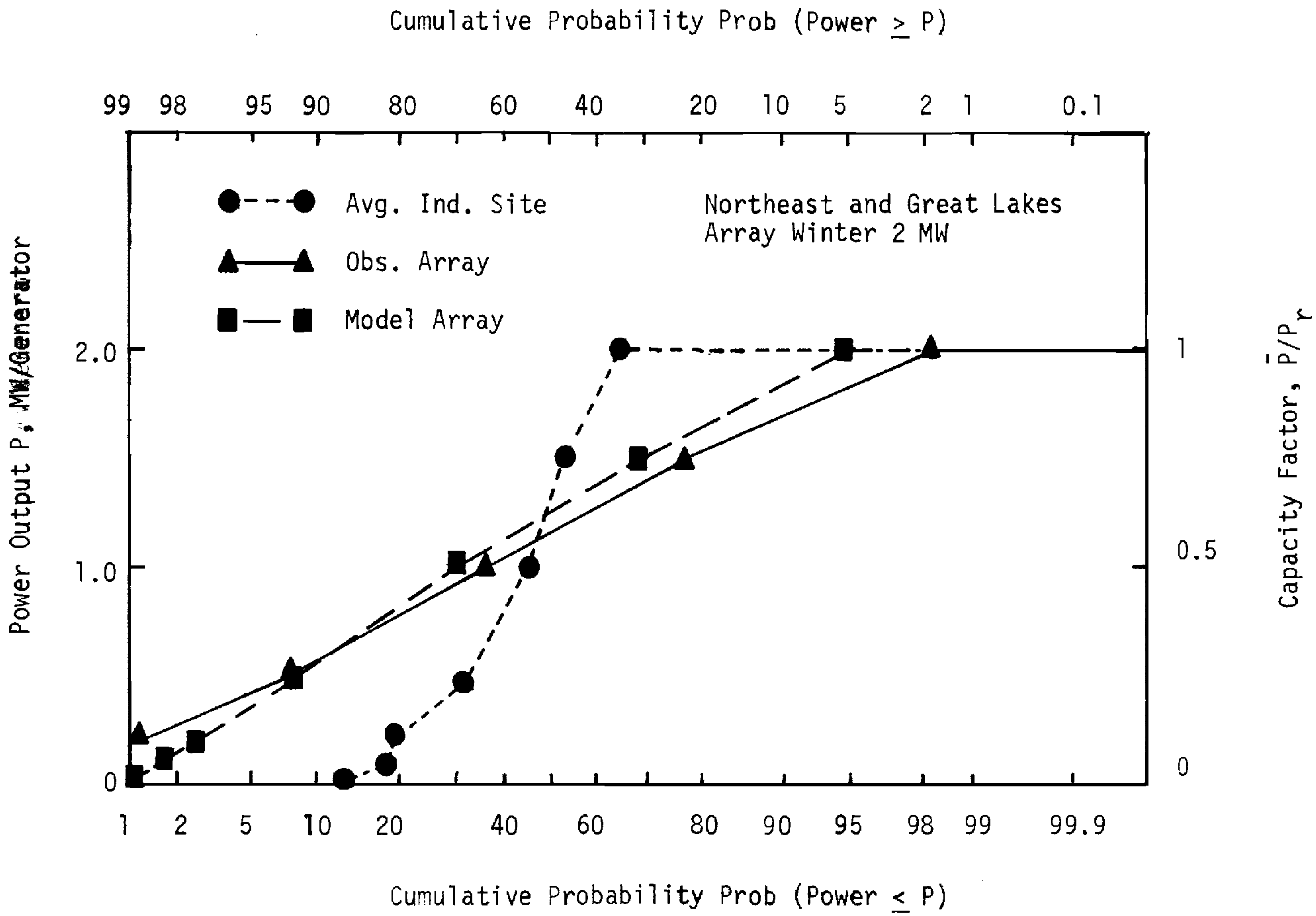

Figure C-1. Cumulative Frequency of Various Power Output Levels for 2 MW WECS. Individual Site and Array Configuration for Northeast and Great Lakes Array.

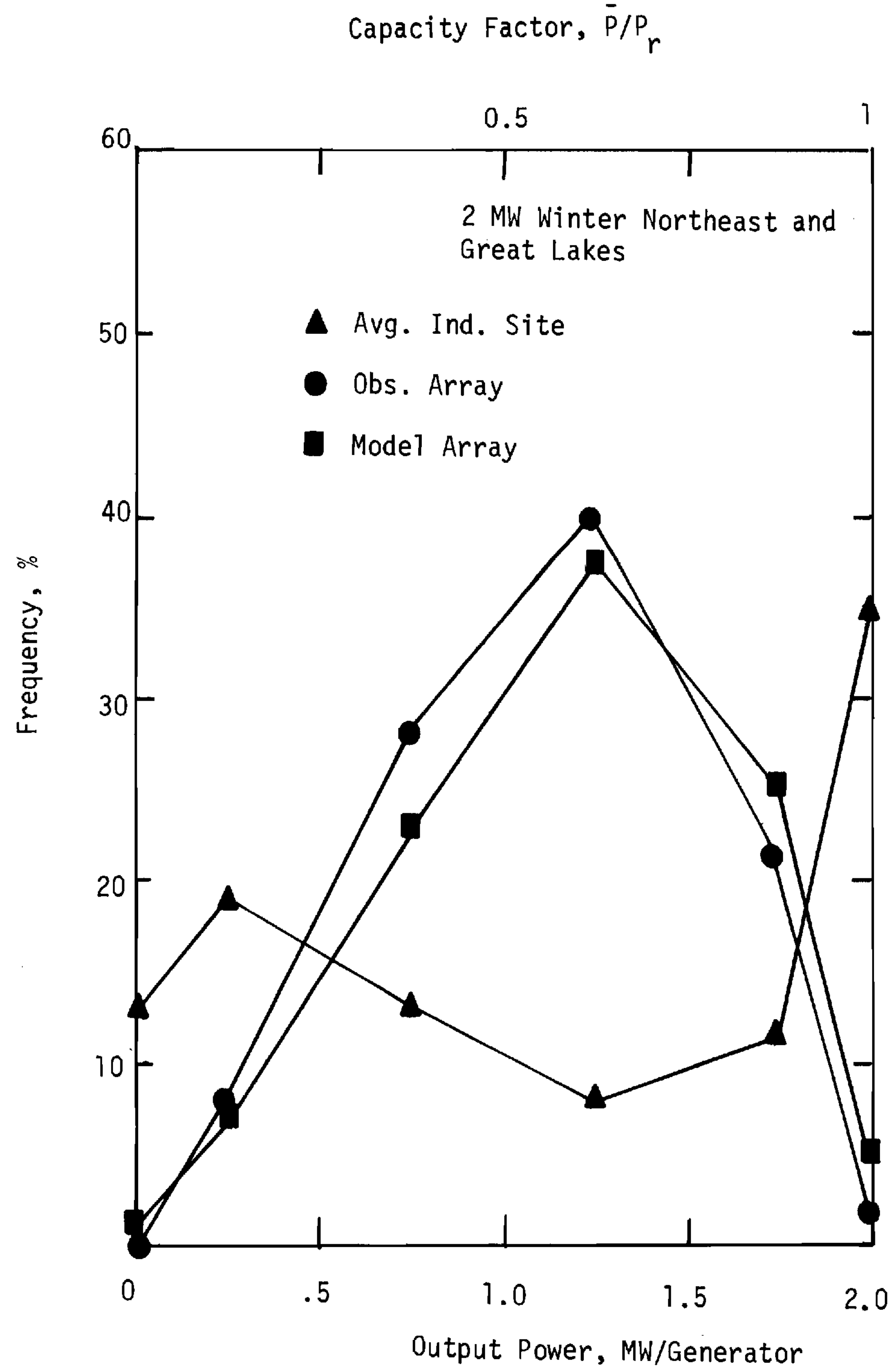

Figure C-2. Frequency of Power Output for 2MW WECS Northeast and Great Lakes Array.

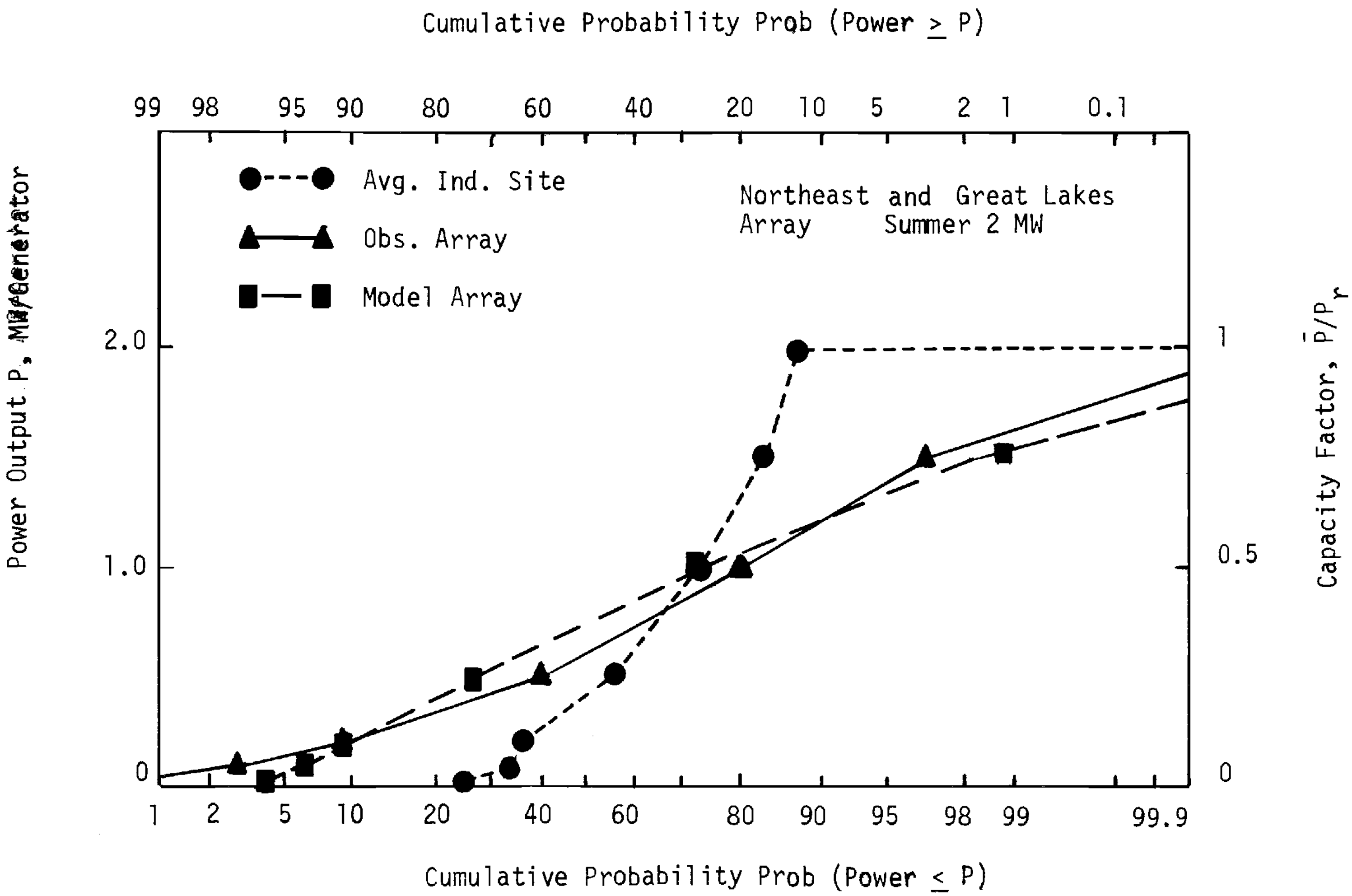

Figure-C-3. Cumulative Frequency of Various Power Output Levels for 2 MW WECS Individual Site and Array Configuration for Northeast and Great Lakes Array.

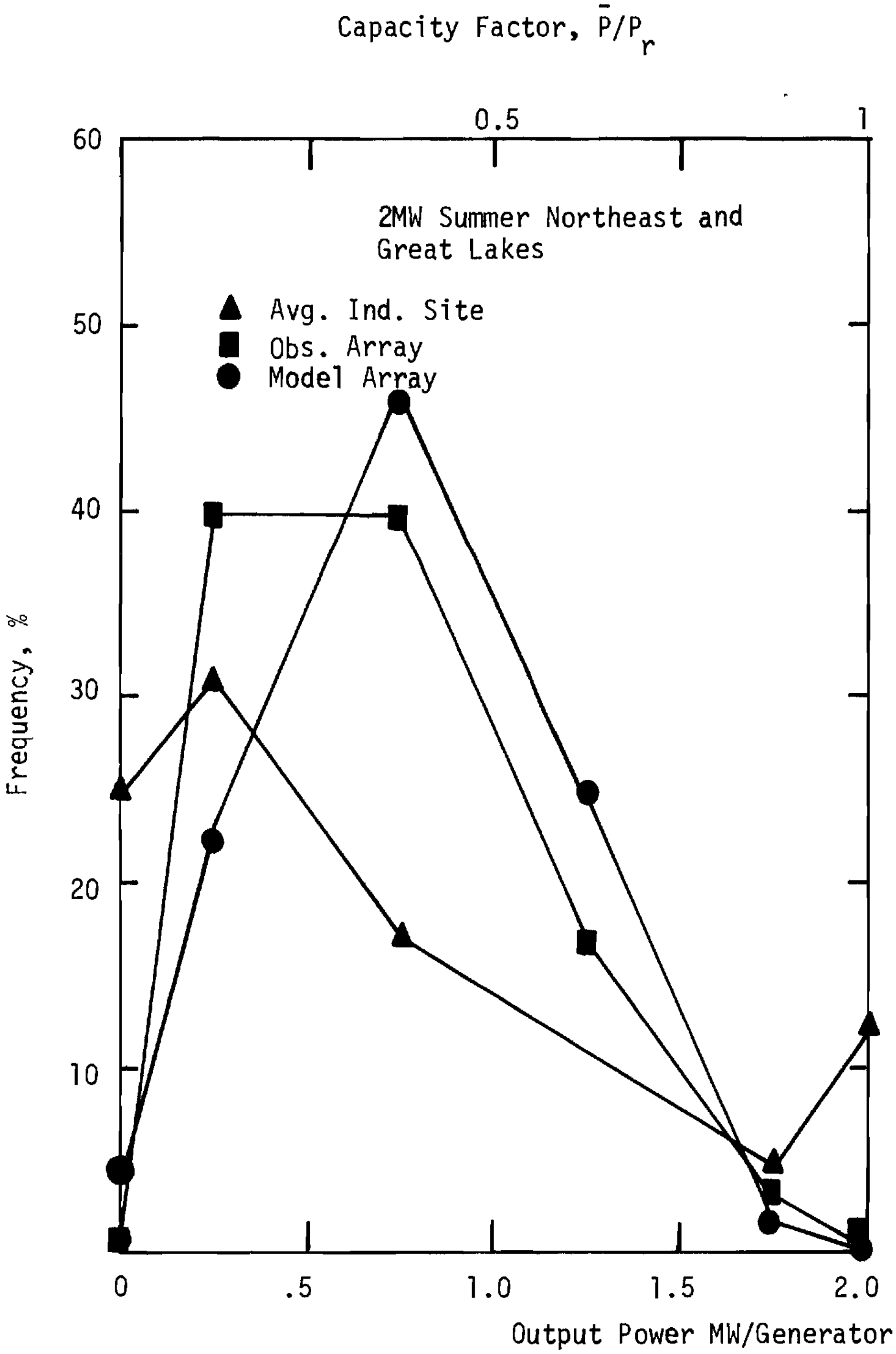

Figure C-4. Frequency of Power Output for 2 MW WECS Northeast and Great Lakes Array

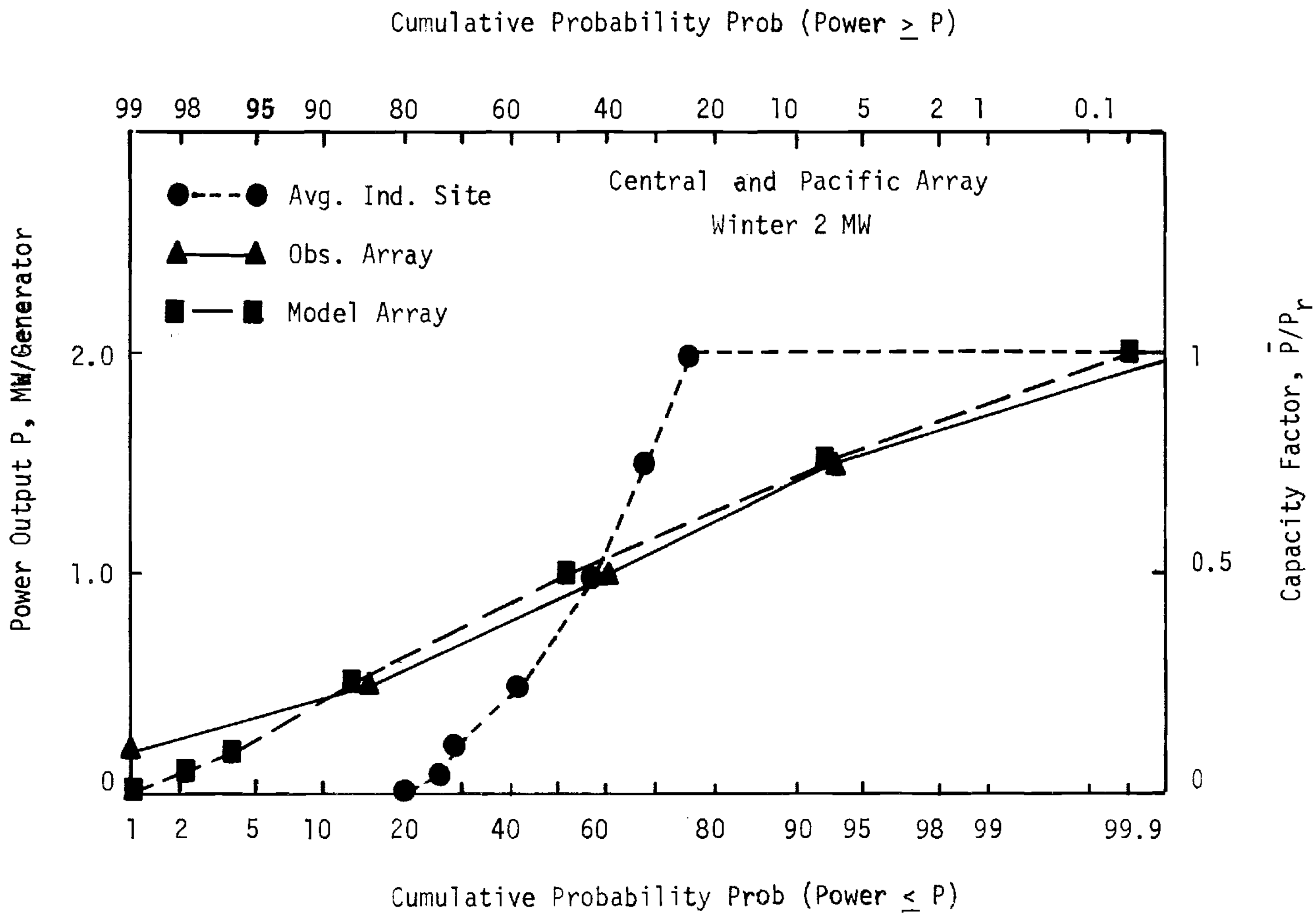

Figure C-5. Cumulative Frequency of Various Power Output Levels for 2 MW WECS Individual Site and Array Configuration for Central and Pacific Array.

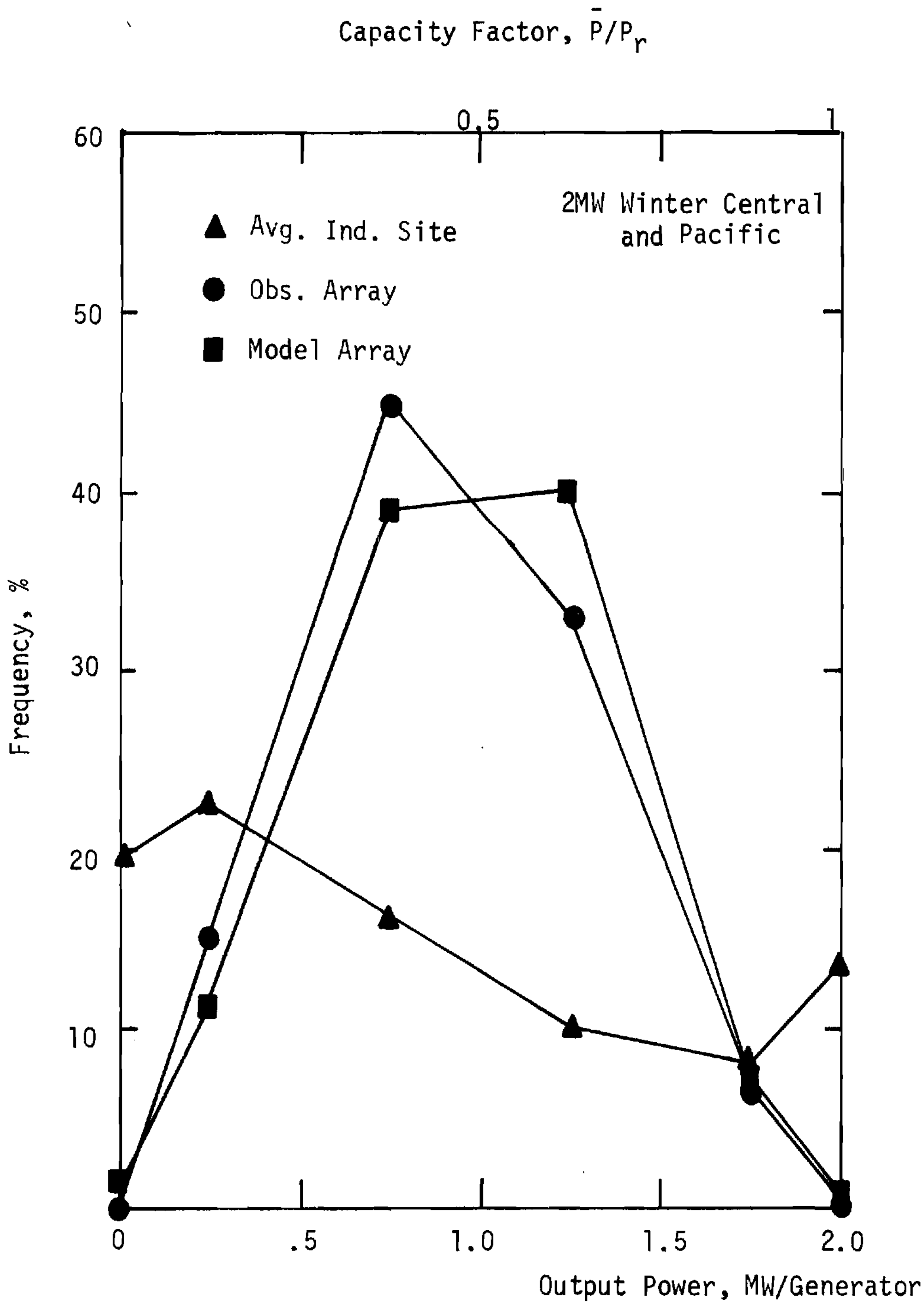

Figure C-6. Frequency of Power Output for 2MW WECS Central and Pacific Array.

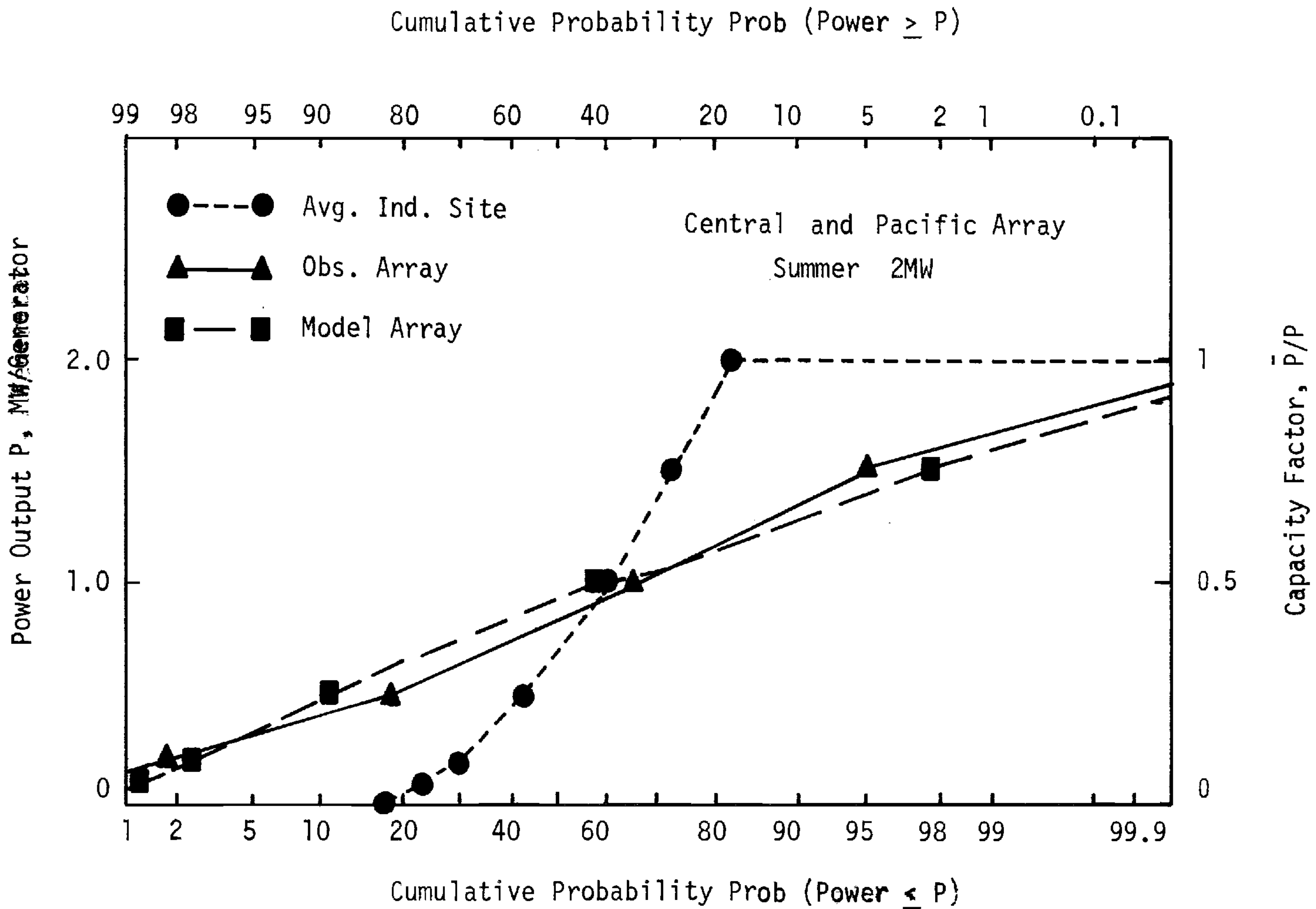

Figure C-7. Cumulative Frequency of Various Output Levels for 2 MW WECS Individual Sites and Array Configurations for Central Pacific Array.

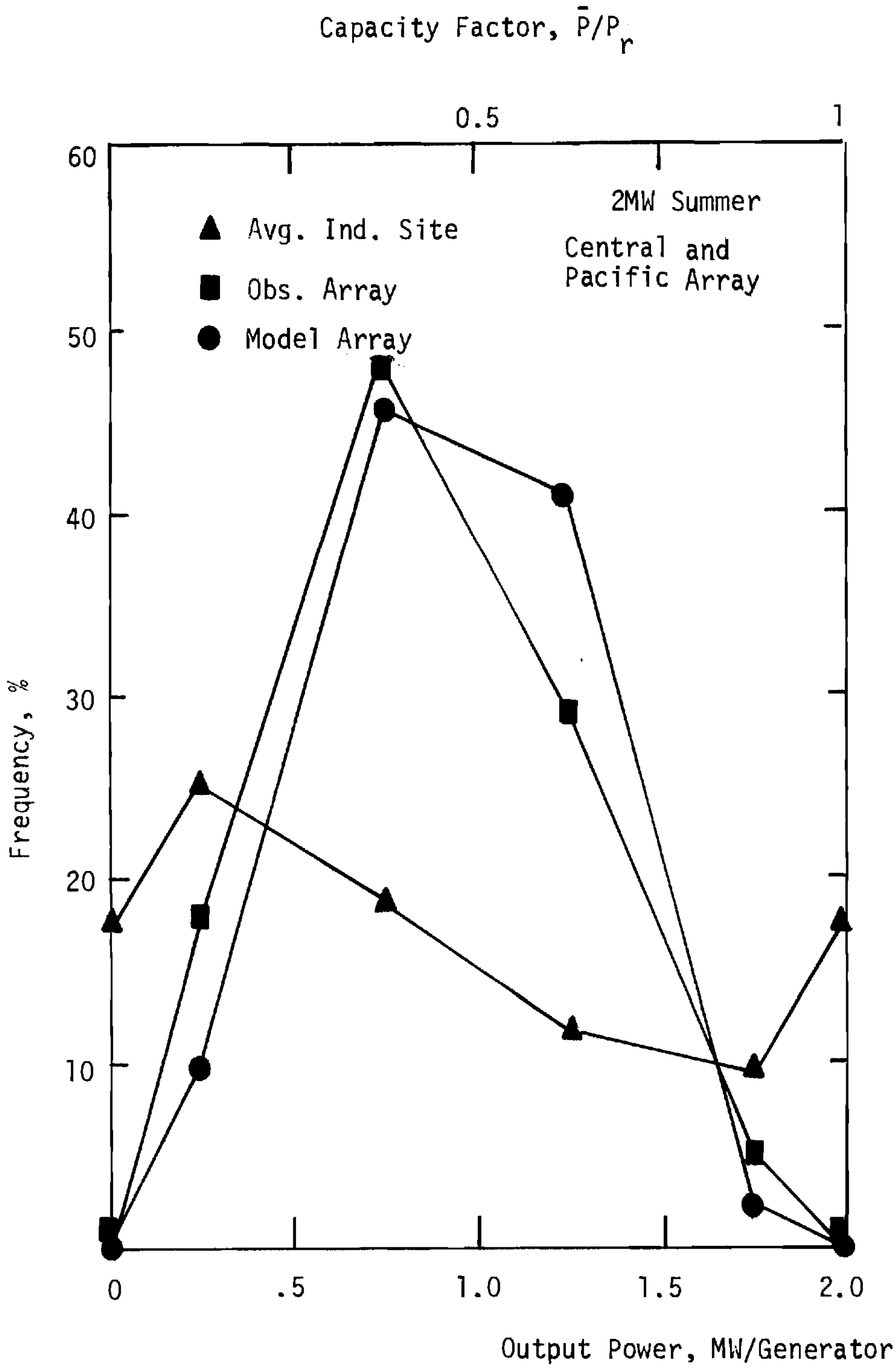

Figure C-8. Frequency of Power Output for 2MW WECS Central and Pacific Array.

Table C-1. Availability (Percent) of 200 kW per 2 MW Generator (10% Capacity Factor) in Northeast and Great Lakes Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	77.4	98.2	67.6	95.7	49.7	83.0	63.6	94.1	64.5	92.7
4	77.0	98.2	65.7	96.4	46.1	81.2	61.0	92.7	62.4	92.1
7	77.2	98.5	71.9	97.1	55.4	87.7	63.8	92.3	67.0	93.9
10	84.3	99.3	83.2	98.9	72.2	96.7	78.1	97.1	79.4	98.0
13	87.7	99.3	90.3	100.0	82.9	99.6	83.2	98.9	86.0	99.5
16	86.6	99.3	91.7	100.0	84.8	100.0	83.3	98.2	86.6	99.4
19	79.0	99.6	81.2	99.6	67.2	97.1	65.3	93.4	73.2	97.4
22	77.9	99.3	68.0	96.7	49.2	81.9	64.7	93.4	64.9	92.8
All	80.9	98.9	77.4	98.1	63.4	90.9	70.4	95.0	73.0	95.7

Table C-2. Availability (Percent) of 500 kW per 2 MW Generator (25% Capacity Factor) in Northeast and Great Lakes Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	64.4	90.4	50.0	77.9	29.9	41.7	44.0	64.5	47.0	68.5
4	62.6	87.8	49.0	72.5	29.1	34.8	42.1	61.9	45.6	64.1
7	63.0	88.6	55.6	80.4	35.2	44.2	44.5	64.5	49.5	69.3
10	72.3	93.7	70.5	94.2	52.8	72.5	61.3	83.2	64.2	85.9
13	76.7	96.7	77.1	98.2	68.0	90.6	67.1	91.2	72.2	94.2
16	74.9	94.8	80.8	100.0	68.9	93.1	64.8	88.3	72.4	94.1
19	64.7	92.6	63.4	89.5	44.3	65.2	45.5	64.5	54.4	77.9
22	63.9	91.1	51.3	76.8	29.8	35.9	45.5	68.1	47.5	67.9
All	67.8	92.0	62.2	86.2	44.7	59.7	51.8	73.3	56.6	77.7

Table C-3. Availability (Percent) of 200 kW per 2 MW Generator (10% Capacity Factor) in Great Lakes and Central Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	78.2	99.3	71.6	98.6	56.1	91.7	65.5	96.7	67.8	96.5
4	78.2	99.3	70.8	97.5	53.0	89.5	64.7	95.6	66.6	95.4
7	77.3	99.3	72.1	98.2	54.4	88.8	65.2	96.3	67.2	95.6
10	82.2	99.6	86.1	100.0	76.6	99.6	79.2	100.0	81.0	99.8
13	87.1	100.0	89.4	100.0	82.2	100.0	83.0	100.0	85.4	100.0
16	85.8	99.6	89.5	100.0	82.2	100.0	82.8	100.0	85.1	99.9
19	75.0	99.6	82.1	99.6	74.2	99.3	65.8	96.3	74.3	98.7
22	77.8	99.6	69.0	96.7	53.9	90.2	65.6	96.3	66.5	95.7
All	80.2	99.5	78.8	98.8	66.6	94.9	71.5	97.7	74.2	97.7

Table C-4. Availability (Percent) of 500 kW per 2 MW Generator (25% Capacity Factor) in Great Lakes and Central Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	63.5	91.5	54.9	84.8	38.7	59.1	46.8	73.6	50.9	77.2
4	62.6	90.0	54.7	78.6	36.6	50.7	46.6	71.8	50.1	72.7
7	62.2	90.4	56.9	81.2	36.7	47.5	46.7	69.2	50.6	72.0
10	67.8	93.0	75.6	97.8	61.6	91.3	64.6	90.5	67.4	93.2
13	75.6	96.7	79.5	99.6	69.7	96.7	70.7	96.0	73.9	97.3
16	74.1	96.3	80.5	98.6	69.7	95.3	68.5	96.0	73.2	96.5
19	59.2	88.6	68.3	96.4	57.9	91.7	48.1	76.2	58.4	88.2
22	60.9	90.4	52.6	79.3	36.4	50.4	46.4	72.2	49.1	73.0
All	65.8	92.1	65.4	89.5	50.9	72.8	54.8	80.7	59.2	83.8

Table C-5. Availability (Percent) of 200 kW per 2 MW Generator (10% Capacity Factor) in Central & Pacific Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	70.3	97.8	74.9	98.9	66.6	99.3	64.2	94.5	69.0	97.6
4	69.7	98.2	72.3	98.6	61.7	96.7	63.1	93.8	66.7	96.8
7	68.7	99.3	69.9	99.3	55.2	91.3	61.8	94.9	63.8	96.2
10	71.4	98.9	76.7	100.0	65.6	98.6	68.3	98.5	70.5	99.0
13	76.0	100.0	81.3	100.0	76.7	100.0	73.0	19.3	77.0	99.8
16	77.7	100.0	86.5	100.0	83.6	100.0	79.1	100.0	81.8	100.0
19	69.3	98.9	85.8	100.0	84.9	100.0	72.2	99.3	78.2	99.5
22	69.9	99.3	75.3	98.9	67.7	99.6	67.1	97.1	70.0	98.7
All	71.8	99.0	77.9	99.5	70.3	98.2	68.6	97.2	72.0	98.5

Table C-6. Availability (Percent) of 500 kW per 2 MW Generator (25% Capacity Factor) in Central & Pacific Array (Individual Sites and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	55.4	81.5	58.9	85.1	51.0	76.4	47.9	72.2	53.3	78.8
4	55.3	83.8	56.3	82.6	44.9	68.8	46.6	66.7	50.7	75.5
7	54.9	83.8	55.2	78.3	38.9	53.6	45.7	65.6	48.6	70.3
10	56.9	84.5	66.2	95.3	53.5	79.3	55.2	78.4	58.0	84.4
13	65.1	94.1	72.9	98.2	64.5	92.8	62.7	90.8	66.3	94.0
16	66.2	94.1	79.9	99.6	74.9	98.9	68.5	95.6	72.5	97.1
19	54.1	78.2	77.2	98.9	75.7	99.6	58.6	86.4	66.6	90.5
22	53.7	80.1	61.2	87.3	54.8	87.7	50.1	71.8	55.0	81.8
All	57.7	85.0	66.0	90.7	57.3	82.2	54.4	78.4	58.9	84.1

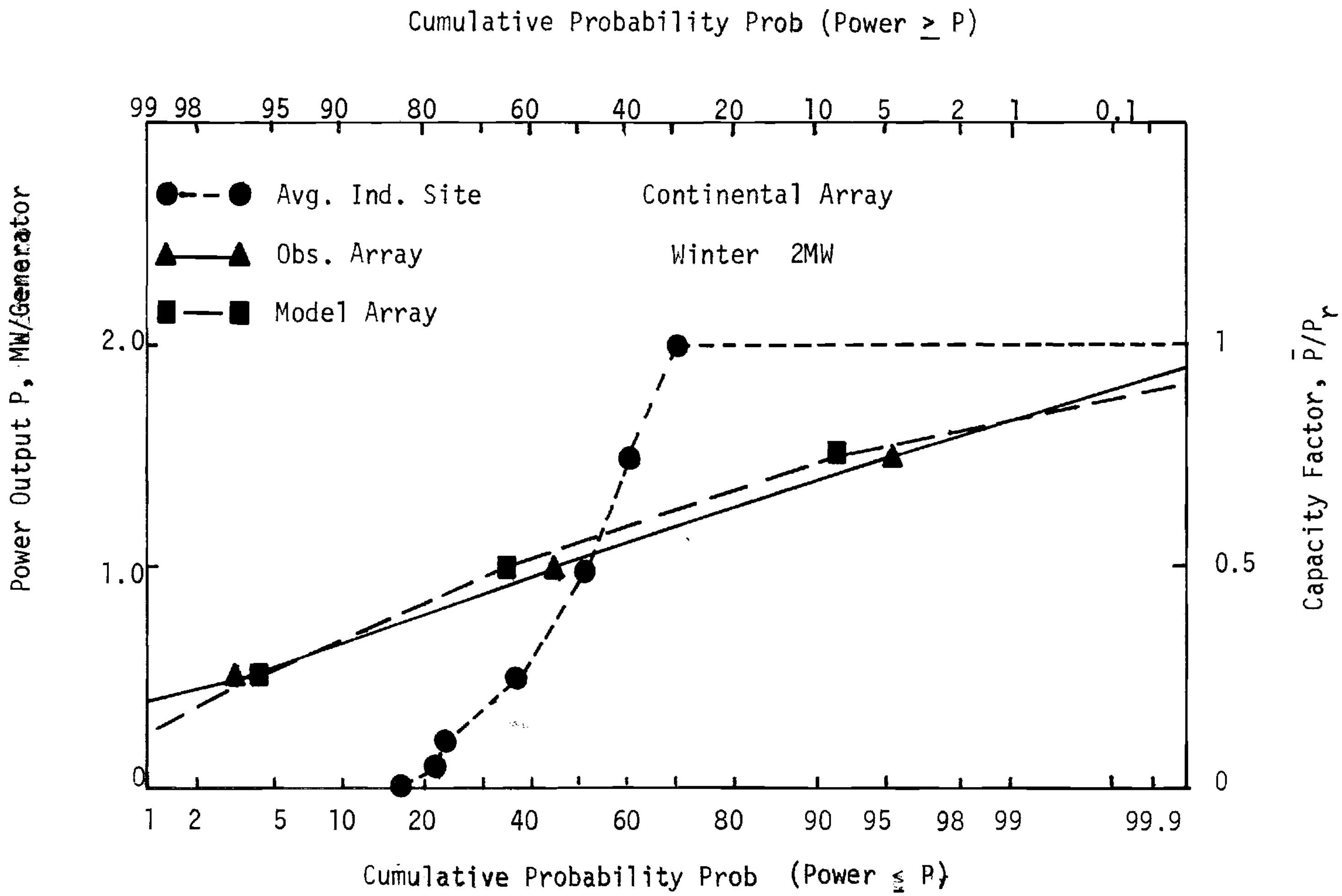

Figure C-9. Cumulative Frequency of Various Output Levels for 2 MW WECS Individual Sites and Array Configurations for Continental Array.

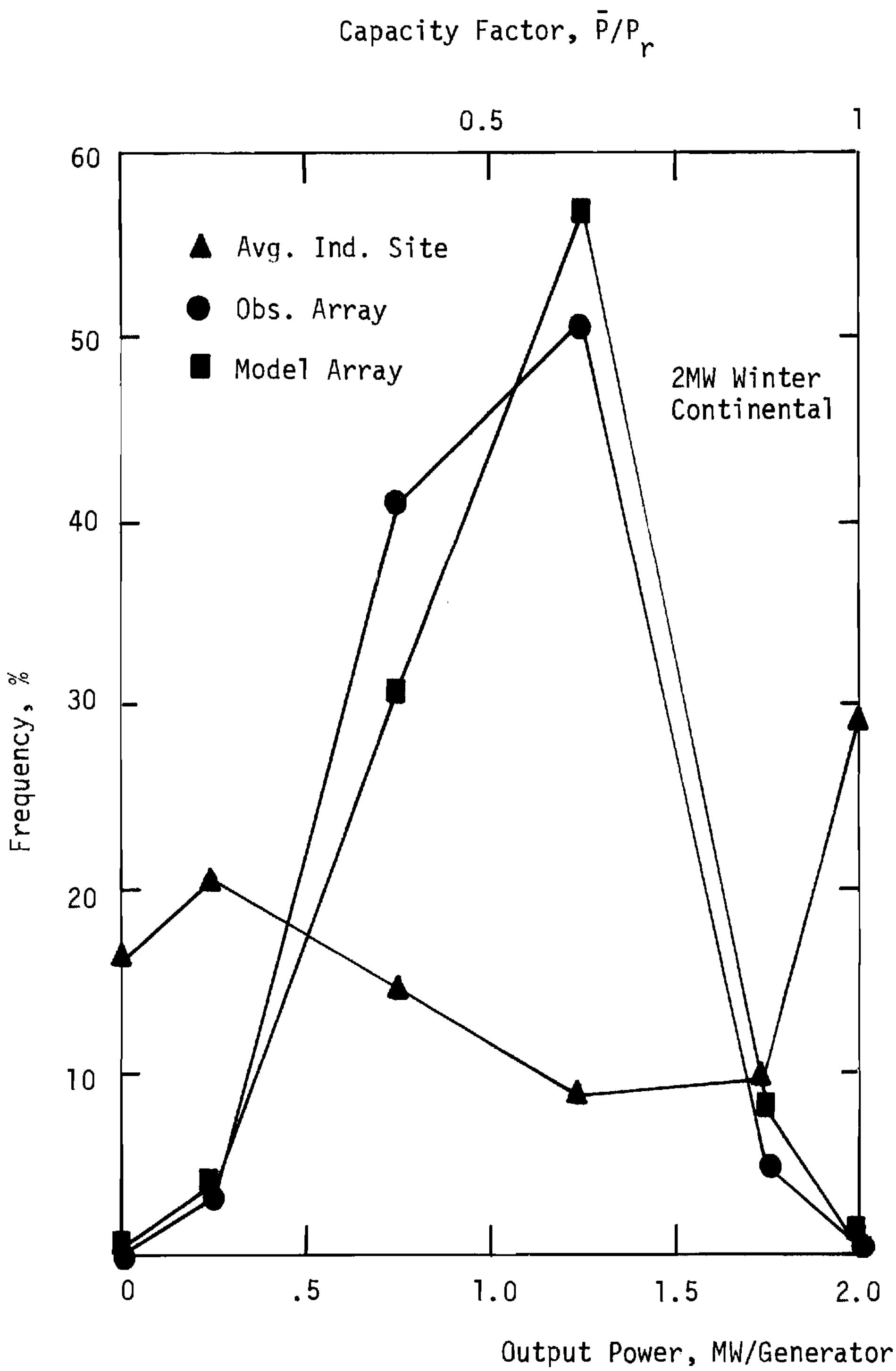

Figure C-10. Frequency of Power Output for 2MW WECS Continental Array.

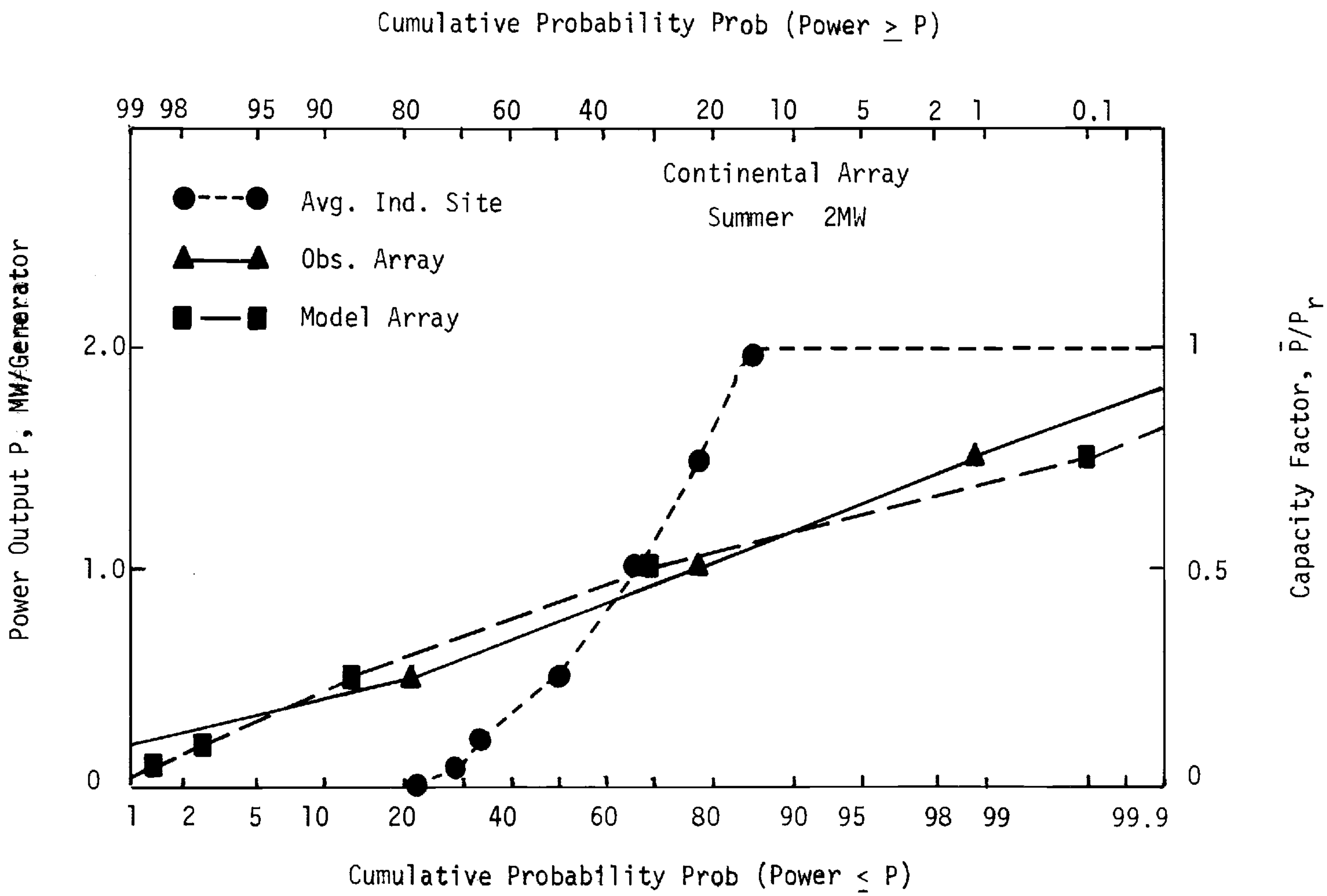

Figure C-11. Cumulative Frequency of Various Output Levels for 2 MW WECS Individual Sites and Array Configurations for Continental Array.

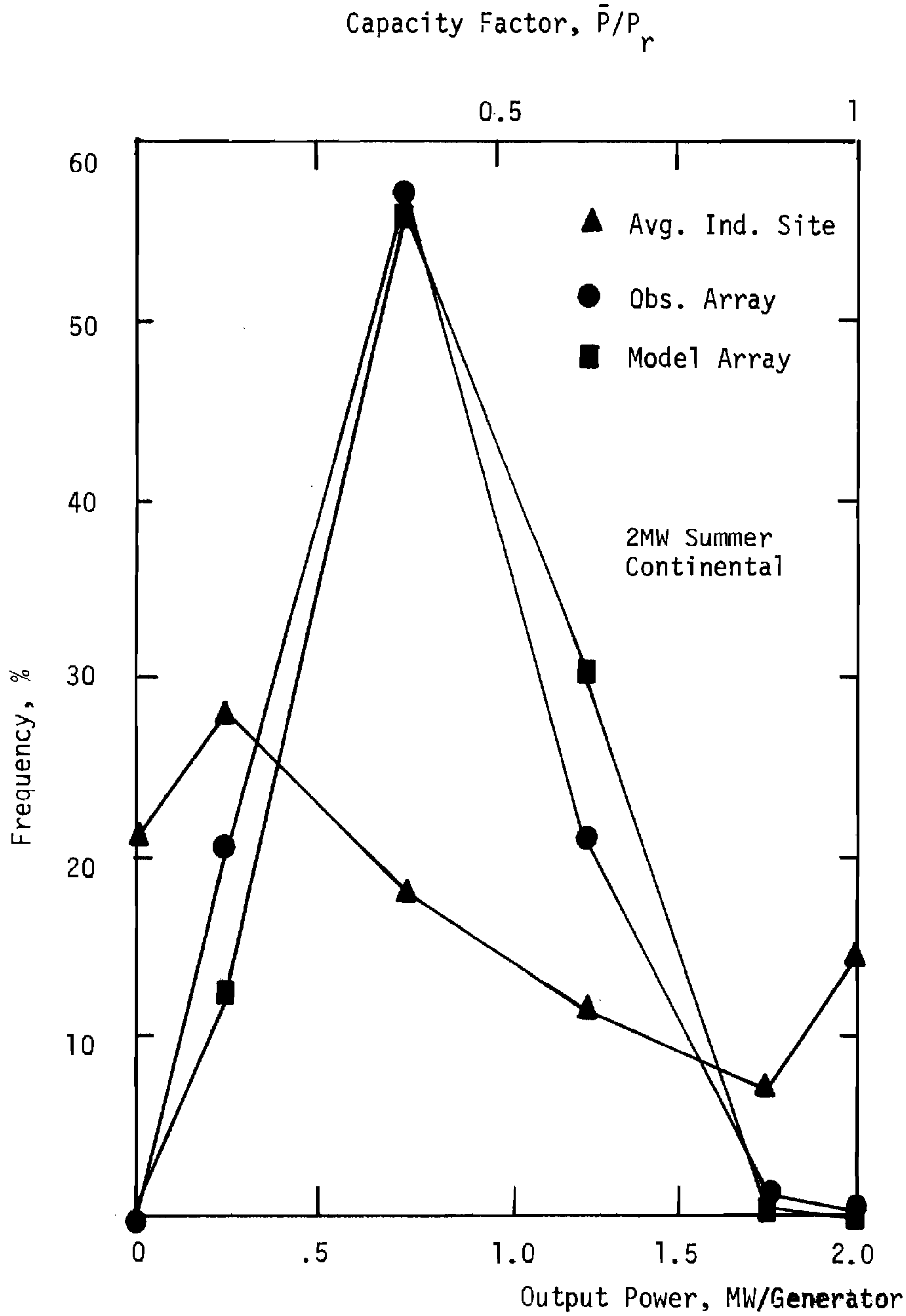

Figure C-12. Frequency of Power Output for 2MW WECS Continental Array.

Table C-7. Availability (Percent) of 200 kW per 2 MW Generator (10% Capacity Factor) in Continental Array (Individual Site and Whole Array)

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	73.9	100.0	71.1	99.6	58.2	99.6	63.7	99.6	66.7	99.7
4	73.3	100.0	68.9	100.0	53.9	98.9	62.0	99.3	64.5	99.5
7	73.0	100.0	70.9	100.0	55.3	97.1	62.7	99.3	65.4	99.1
10	78.0	100.0	80.0	100.0	68.9	99.6	73.3	100.0	75.0	99.9
13	82.4	100.0	85.8	100.0	79.8	100.0	78.2	100.0	81.6	100.0
16	82.2	99.6	89.1	100.0	84.2	100.0	81.1	100.0	84.2	99.9
19	74.2	100.0	83.4	100.0	76.1	100.0	68.5	100.0	75.6	100.0
22	73.9	100.0	71.6	99.6	58.5	100.0	65.8	100.0	67.4	99.9
All	76.4	100.0	77.6	99.9	66.9	99.4	69.4	99.8	72.5	99.8

Table C-8. As in Table C-7 for 500 kW per 2 MW Generator (25% Capacity Factor) in Continental Array

	Winter		Spring		Summer		Fall		Annual	
Hour	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array	Ind.	Array
1	60.0	95.2	54.4	89.5	40.4	68.8	45.9	82.1	50.1	83.9
4	58.9	94.5	52.6	87.3	37.0	52.9	44.3	72.9	48.1	76.8
7	59.0	94.8	55.4	88.4	37.1	52.9	45.1	74.7	49.0	77.6
10	64.7	97.4	68.4	98.2	53.2	88.0	58.3	91.2	61.1	93.7
13	70.9	98.9	75.0	100.0	66.2	99.3	65.0	98.2	69.3	99.1
16	70.6	98.2	80.4	100.0	71.9	99.3	66.6	97.4	72.4	98.7
19	59.4	97.0	70.2	99.6	60.0	99.3	51.9	90.5	60.4	96.6
22	58.9	97.8	56.2	91.3	42.3	73.2	47.7	82.1	51.2	86.0
All	62.8	96.7	64.1	94.3	51.0	79.2	53.1	86.1	57.7	89.1

APPENDIX D

WIND POWER PERFORMANCE EVALUATION MODEL

The Power Coefficient Model

The output power curves for a very wide range of constant RPM wind turbine designs can be represented by the following power coefficient relation versus hub height wind speed V (Justus, 1978)

$$C_p(V)/C_{pm} = \begin{cases} 0 & V < V_{in} \\ 1 - A(V_m/V-1)^2 - B(V_m/V-1)^3 & V_{in} \leq V \leq V_r \\ (C_{pr}/C_{pm})(V_r/V)^3 & V_r < V \leq V_{out} \\ 0 & V > V_{out} \end{cases} \tag{D-1}$$

where C_{pm} is the maximum power coefficient, achieved at the design hub height wind speed V_m; V_{in} is the hub height cut-in wind speed; V_r is the hub height rated wind speed, at which the power coefficient is C_{pr}; V_{out} is the cutout speed at hub-height; and A and B are coefficients, which can be determined by solving for the conditions

$$\begin{aligned} C_p(V_{in})/C_{pm} &= 1 - A(V_m/V_{in} - 1)^2 - B(V_m/V_{in} - 1)^3 = 0 \\ C_p(V_r)/C_{pm} &= 1 - A(V_m/V_r - 1)^2 - B(V_m/V_r - 1)^3 = C_{pr}/C_{pm} \end{aligned} \tag{D-2}$$

In the special case of $V_m = V_r$, $C_{pm} = C_{pr}$, the solutions $A = 0$, $B = 1/(V_r/V_{in} - 1)^3$ may be used. In general, the solutions for equation D-2 are, in terms of $R_r = V_m/V_r - 1$ and $R_{in} = V_m/V_{in} - 1$,

$$\begin{aligned} A &= [R_r^3 - (1 - C_{pr}/C_{pm})R_{in}^3]/D \\ B &= [R_{in}^2(1 - C_{pr}/C_{pm}) - R_r^2]/D \end{aligned} \tag{D-3}$$

where

$$D = R_{in}^2 \, R_r^2(R_r - R_{in}) \tag{D-4}$$

The effects of power train losses and electrical efficiencies are included in D-1; hence, electrical power output P(V) at wind speed V, relative to rated power P_r (achieved at V_r) is given by

$$P \quad P(V)/P_r = \begin{cases} 0 & V < V_{in} \\ (C_{pm}/C_{pr})(V/V_r)^3[1 - A(V_m/V-1)^2 - B(V_m/V-1)^3] & \\ & V_{in} \leq V \leq V_r \\ 1 & V_r < V \leq V_{out} \\ 0 & V > V_{out} \end{cases} \tag{D-5}$$

For constant rpm WECS designs V_{in}/V_r is usually in the range of 0.4 to 0.5, and C_{pm}/C_{pr} is usually in the range of about 0.7 to 1. For variable rpm WECS, which vary rpm to maintain power coefficient at or near C_{pm} at all wind speeds above cut-in, this model can still be used, especially if V_{in}/V_r is small, and one takes $V_m = V_r$, $C_{pm} = C_{pr}$. Further discussion of modeling for variable rpm machines is given by Justus and Mikhail (1978). They show that average power out of variable rpm machines is quite similar to that from constant rpm machines with $C_{pr} = C_{pm}$.

Variable rpm machines would still have mechanical and electrical power losses near wind speed V_{in}, which would drop the effective C_p below C_{pm}. Note also that the concept of rated speed for variable rpm machines is valid because rated power for a particular design will be specified by the rating of the generator used in the WECS. Figure D-1 shows a plot of C_p/C_{pr} versus V/V_r for various V_{in}/V_r ratios, with

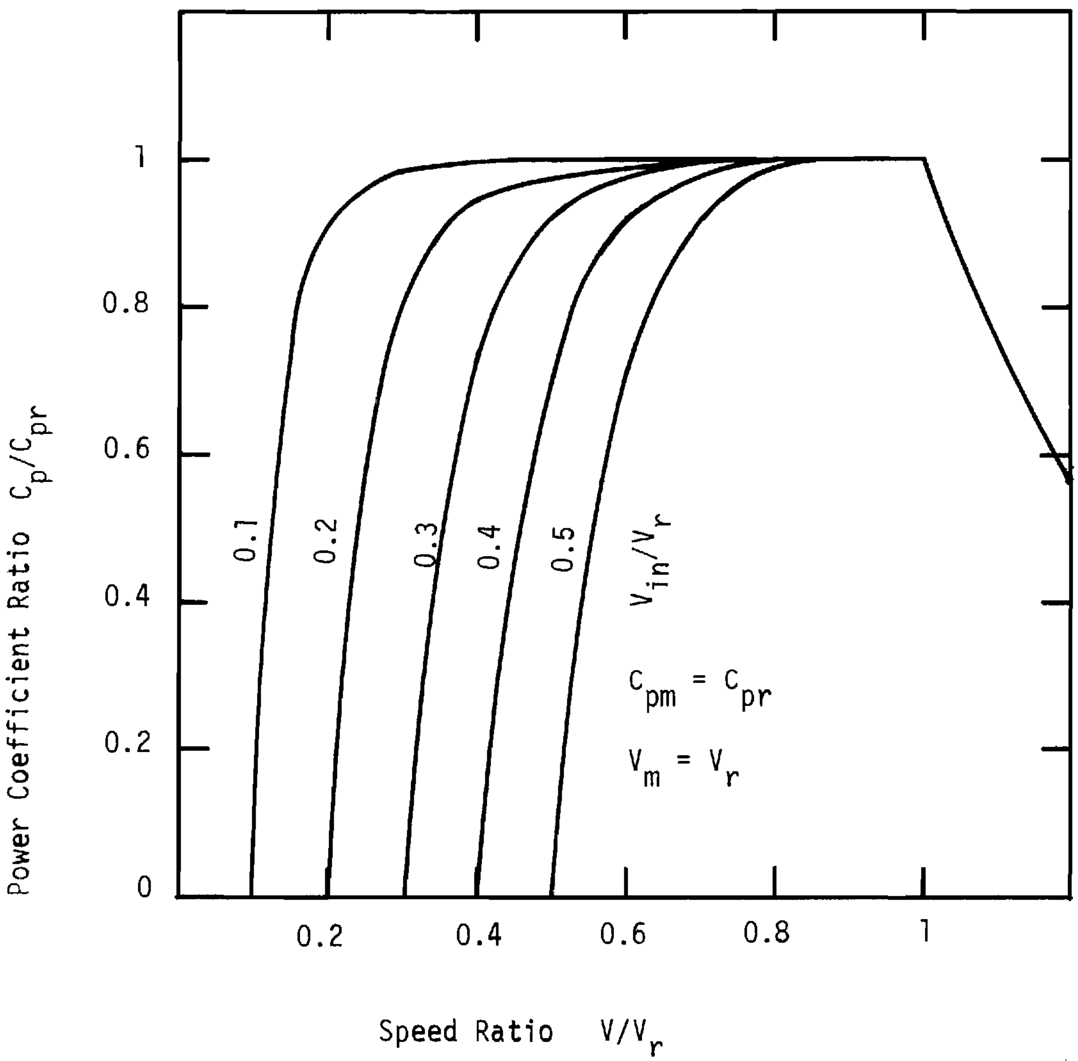

Figure D-1. Relative Power Coefficient C_p/C_{pr} versus V/V_r.

$C_{pr} = C_{pm}$.

Power Performance Evaluation

A range of WECS designs (different values for V_{in}/V_r, V_m/V_r, C_{pr}/C_{pm}, V_{out}/V_r) have been run through the set of reference wind speed distributions (Justus et al, 1976), which have reference low, medium, and high variance Weibull distribution statistics. Power output, expressed as capacity factor $\bar{P}/P_r$, was evaluated for each reference Weibull distribution p(V) by numerical evaluation of the integral

$$\bar{P}/P_r = \int_0^{\infty}[P(V)/P_r]p(V)dV \qquad \text{(D-6)}$$

where P(V) is the power function from equation D-5. The resulting capacity factor values were found to vary significantly with the hub height wind speed ratio $\bar{V}/V_r$, with the power coefficient ratio C_{pr}/C_{pm}, with the variance (Weibull shape parameter) of the distribution, with the cut-in speed ratio V_{in}/V_r, and with the cutout speed ratio V_{out}/V_r. No significant variation with the design speed ratio V_m/V_r was found. This is a somewhat surprising result, since both C_{pr}/C_{pm} and V_m/V_r might be considered, a priori, as important parameters. In fact, only the maximum power coefficient value (C_{pr}/C_{pm}) is important, not the speed where it occurs (V_m/V_r). Over the range of average wind speeds $0.4 \leq \bar{V}/V_r \leq 1$, the capacity factor was found to be quite linear with respect to $\bar{V}/V_r$. Multiple linear regression of the form

$$\bar{P}/P_r = \alpha_0 + \alpha_1\,(V_{in}/V_r) + \alpha_2\,(C_{pr}/C_{pm}) + \alpha_3\,(\bar{V}/V_r) \qquad \text{(D-7)}$$

was performed on the capacity factor values, separately for each variance condition and each V_{out}/V_r value. Table D-1 gives values for α_0 through α_3 for the various V_{out}/V_r ratios examined for the low, average, and high variance reference distributions.

Table D-1. Linear Approximation Capacity Factor Regression Parameters [equation D-7] for the Limited Range of Hub Height Average to Rated Speed Ratio $0.4 \le \bar{V}/V_r \le 1$.

Cutout Speed Ratio V_{out}/V_r	Variance	α_0 (const. term)	α_1 (V_{in}/V_r term)	α_2 (C_{pr}/C_{pm} term)	α_3 ($\bar{V}/V_r$ term)	rms error %
		Coefficients in Equation D-7				
1.0	low	+0.427	-0.190	-0.236	0.175	3.3
	avg.	+0.409	-0.182	-0.219	0.152	2.8
	high	+0.358	-0.161	-0.181	0.106	2.0
1.5	low	-0.035	-0.190	-0.236	1.054	2.4
	avg.	+0.019	-0.182	-0.219	0.941	2.6
	high	+0.142	-0.161	-0.181	0.660	2.6
2.0	low	-0.107	-0.190	-0.236	1.170	1.4
	avg.	-0.074	-0.182	-0.219	1.095	1.4
	high	+0.016	-0.161	-0.181	0.888	1.6
2.5	low	-0.109	-0.190	-0.236	1.172	1.4
	avg.	-0.078	-0.182	-0.219	1.102	1.3
	high	-0.008	-0.161	-0.181	0.927	1.2

In addition to the linear equation D-7 over the limited $\bar{V}/V_r$ range, a nonlinear regression was done over the full $\bar{V}/V_r$ range. Figures D-2 through D-4 show results of these nonlinear regression analyses and demonstrate the sensitivity of capacity factor to C_{pr}/C_{pm} ratio, to variance of the distribution, and to cut-out speed ratio V_{out}/V_r, respectively. Since the V_{in}/V_r coefficient (α_1) and C_{pr}/C_{pm} coefficient (α_2) are approximately equal (see Table D-1), the change of V_{in}/V_r from 0.5 to 0.35 (30% change) would result in similar changes in capacity factor to those produced by the change in C_{pr}/C_{pm} from 1.0 to 0.7, (30% change) shown in Figure D-2.

In Figure D-3, in the range $0.6 \leq \bar{V}/V_r \leq 0.7$, the capacity factor is not sensitive to the variance (Weibull shape parameter) of the distribution. In the power performance calculations of Cliff (1977), he used k = 2 (Rayleigh distribution). Figure D-5, which contains the original k versus $\bar{V}$ reference data, as well as newer results combining additional data from a large number of sites (mostly from Verholek, 1977), shows that the Weibull k value of 2 is only representative of $\bar{V} \simeq 4.5$ m/s. However, any Weibull k value could be used in $0.6 \leq \bar{V}/V_r \leq 0.7$ range with no loss in accuracy of capacity factor estimates. Outside this $\bar{V}/V_r$ range k values from Figure D-5 are recommended.

Figures D-2 through D-4 all show that the linear relation in $\bar{V}/V_r$ equation D-7 gives a very good approximation to the nonlinear $\bar{V}/V_r$ values in the range $0.4 \leq \bar{V}/V_r \leq 1$. Thus, for a given WECS design (specific values for V_{in}/V_r, C_{pr}/C_{pm}, V_{out}/V_r) the capacity factor versus $\bar{V}/V_r$ can be expressed in the simple form

$$\bar{P}/P_r = a + b\,(\bar{V}/V_r) \qquad \text{(D-8)}$$

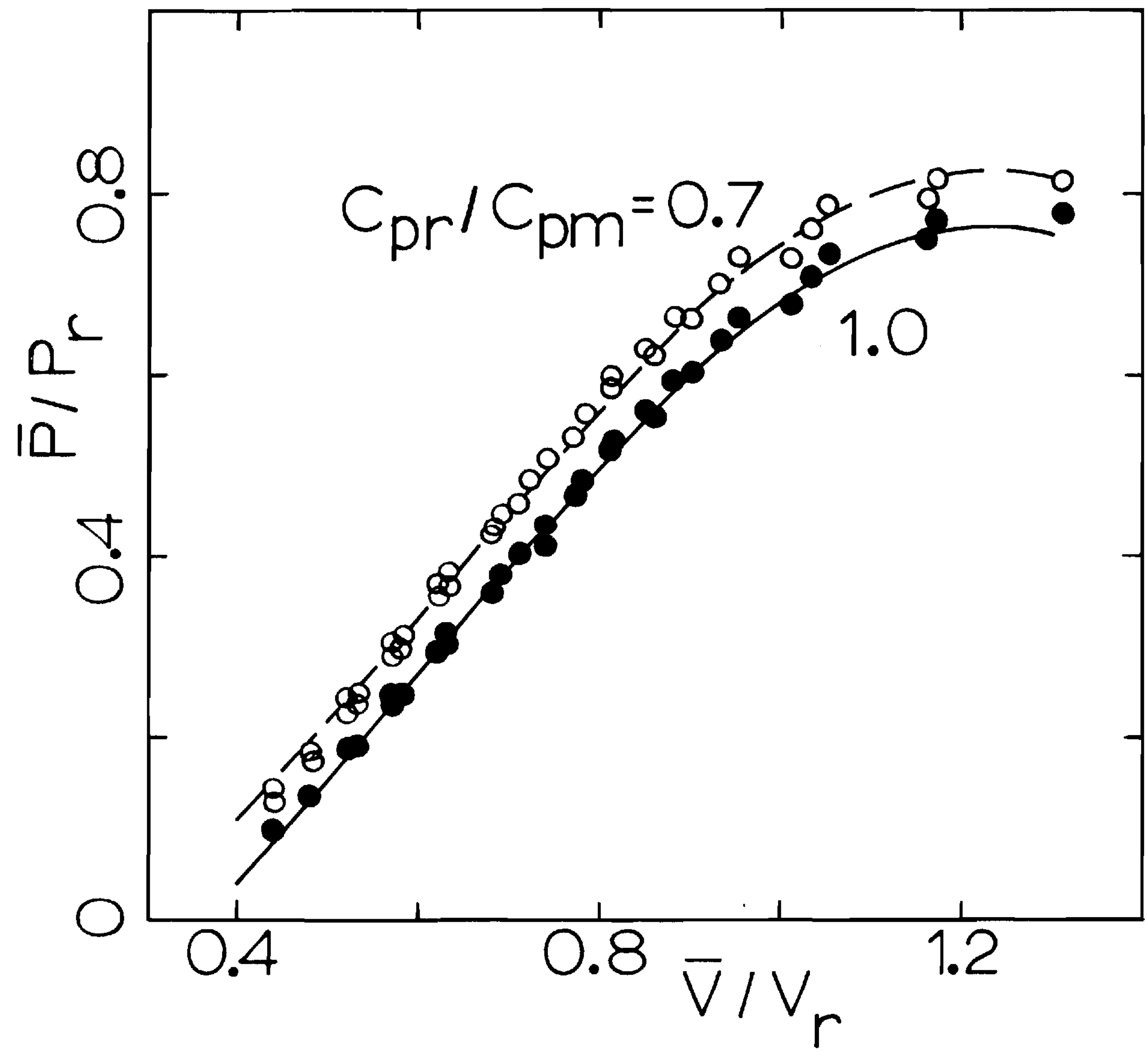

Figure D-2. Dependence of the Capacity Factor $\bar{P}/P_r$ on the Speed Ratio $\bar{V}/V_r$ and the Power Coefficient Ratio C_{pr}/C_{pm} (average variance, $V_{in}/V_r = 0.5$, $V_{out}/V_r = 2$)

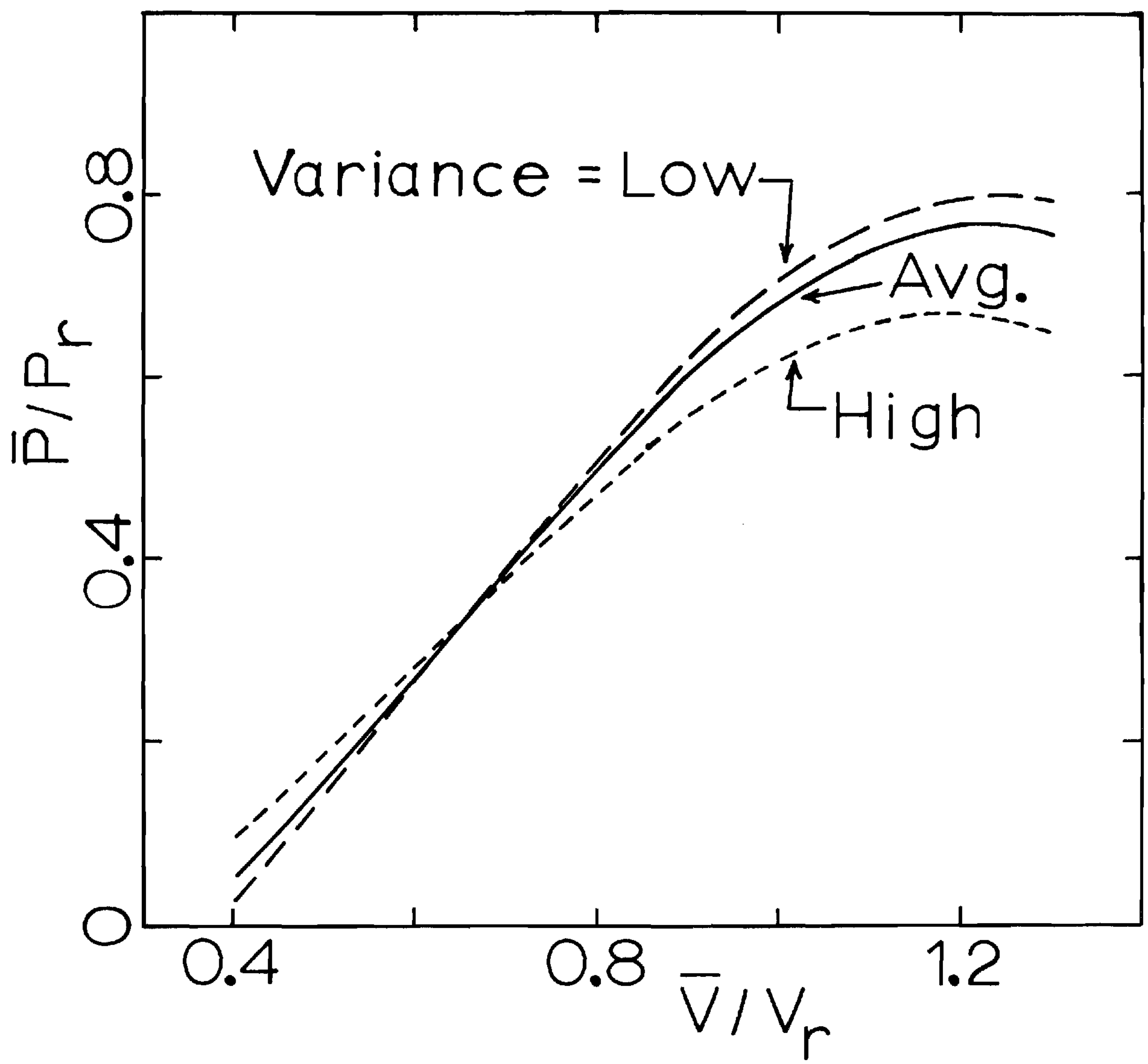

Figure D-3. Dependence of the Capacity Factor $\bar{P}/P_r$ on the Speed Ratio $\bar{V}/V_r$ and the Variance of the Wind Speed Distribution (C_{pr}/C_{pm} = 1, V_{in}/V_r = 0.5, V_{out}/V_r = 2)

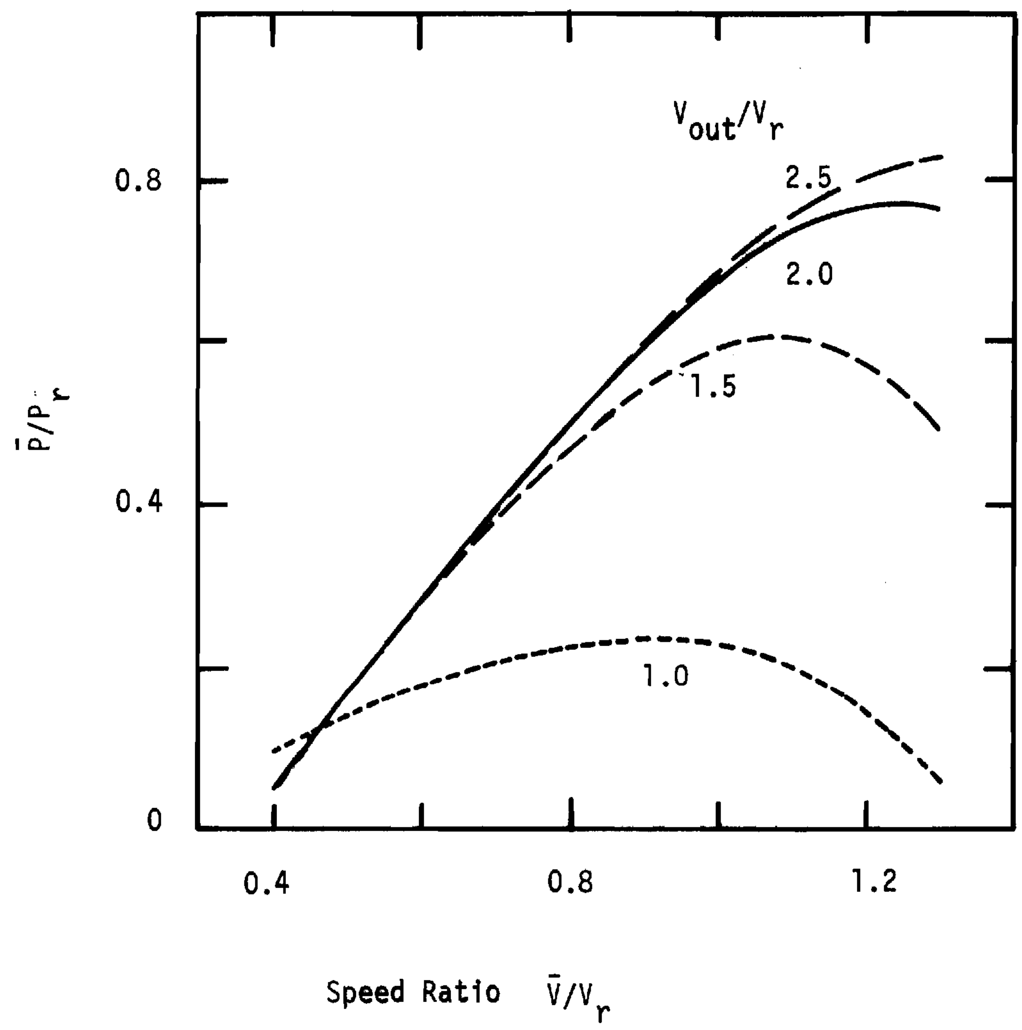

Figure D-4. Dependence of the Capacity Factor $\bar{P}/P_r$ on the Cutout Speed Ratio V_{out}/V_r (C_{pr}/C_{pm}=1, V_{in}/V_r = 0.5, Average Variance).

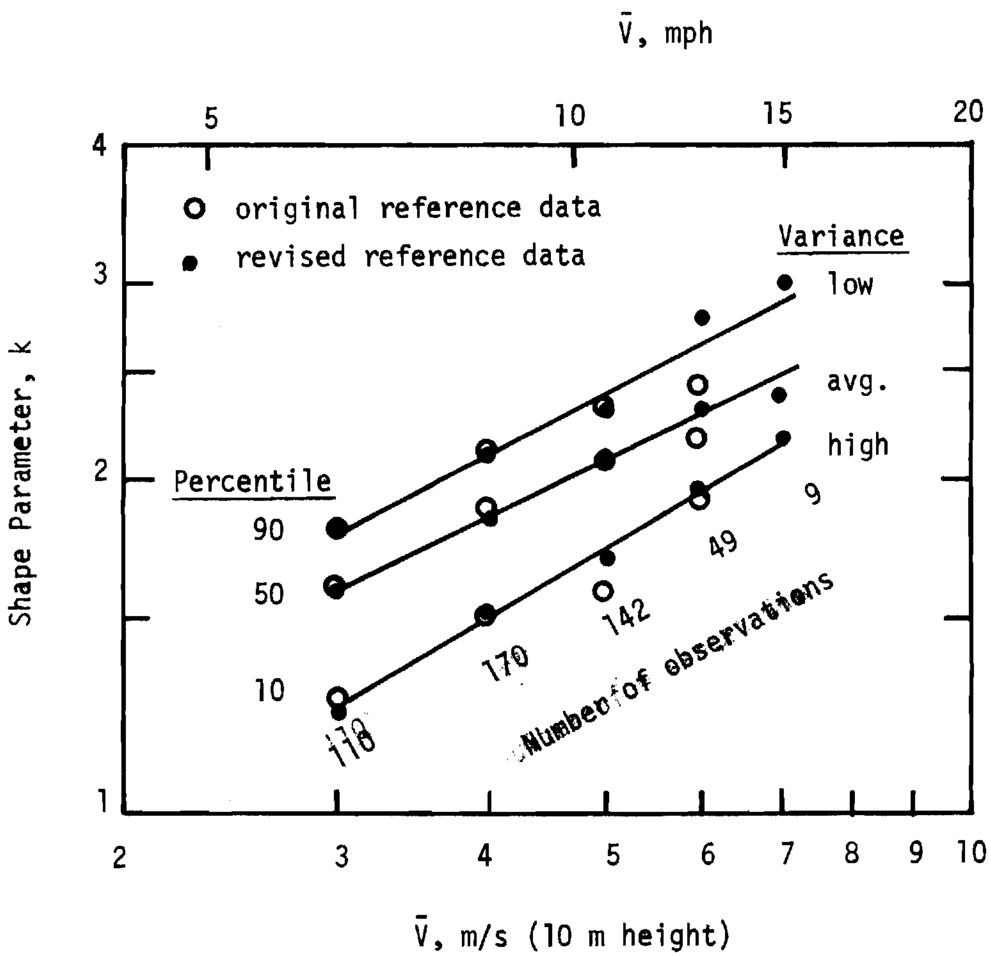

Figure D-5. Revised Reference Statistics of Weibull Shape Parameter k Versus 10 m Mean Wind Speed V̄.

where a and b are determined from the appropriate set of α values from Table D-1. As a specific example, consider the Mod 0-A unit [P_r = 200 kW, V_r = 10.0 m/s (22.4 mph), V_{in} = 4.2 m/s (9.5 mph), V_{out} = 17.9 m/s (40 mph)]. From the design power curve for this machine (NASA, 1978), it is estimated that the appropriate values for V_m/V_r and C_{pr}/C_{pm} are 0.66 and 0.74, respectively. If V_{out}/V_r = 17.9/10.0 = 1.79 is used, the α coefficients for average variance from Table D-1 are α_0 = -0.035, α_1 = -0.182, α_2 = -0.219, α_3 = 1.030. With the values V_{in}/V_r = 4.2/10.0 = 0.42 and C_{pr}/C_{pm} = 0.66, the value of a in equation D-8 would become

$$a = \alpha_0 + \alpha_1 (V_{in}/V_r) + \alpha_2 (C_{pr}/C_{pm})$$

$$= -0.035 - 0.182 (0.42) - 0.219 (0.66) = -0.256.$$

The b value for equation D-8 is the α_3 value (1.030). Thus, for this example with the 200 kW machine, equation D-8 for average power $\bar{P}$ (in kW) versus average speed $\bar{V}$ (in m/s) becomes

$$\bar{P} = P_r (a + b \bar{V}/V_r)$$

$$= 200 (-0.256 + 1.030 \bar{V}/10.0)$$

$$= -51.2 + 20.6 \bar{V}$$

Applications for Array Power Performance

Equation D-8 was developed for estimation of time average (e.g. monthly or annual) power output versus time average wind speed. However, the array studies (Justus, 1976; Justus and Hargraves, 1977a) have shown that the spatial averaging effect of a WECS array at a given time is, in the statistical sense, analogous to the time averaging process. It has been confirmed in these array studies that array power $P(V_{array})$ versus array average speed V_{array} at any time is adequately modeled by the same linear regression relations (with approximately the same numerical coefficient values) as the time average case for a single machine, i.e. the array capacity factor (power per generator in the array) at a given time when wind speed averaged across the array is V_{array}, is given by

$$P(V_{array})/P_r = a + b\ (V_{array}/V_r) \tag{D-9}$$

and the same numerical coefficients a and b can be used in equation D-9 as in equation D-8. Evaluation of equation D-8 for $P(V_{array}) = 0$ and $P(V_{array}) = P_r$ shows effective array "cut-in" and "rated" speeds

$$\begin{aligned} V_{array}(in) &= -\ aV_r/b \\ V_{array}(rated) &= (1 - a)V_r/b \end{aligned} \tag{D-10}$$

[for the 200 kW machine example $V_{array}(in) = 0.236(10)/1.03 = 2.3$ m/s (5 mph) and $V_{array}(out) = 1.236(10)/1.03 = 12$ m/s (26.8 mph)]. Of course, it is not valid to apply equation D-8 all the way down to $P(V_{array}) = 0$ or up to $P(V_{array}) = P_r$ because the linear model is restricted to the range $0.4 \leq V_{array}/V_r \leq 1$ [i.e. for the 200 kW machine example $4 \leq V_{array} \leq 10$ m/s]. However, the probability distribution $p(V_{array})$ is considerably narrower than individual site speed distributions and the error in the product $p(V_{array})\ P(V_{array})/P_r$ [e.g. as would be used in evaluation of mean array output power

$$\bar{P}_{array}/P_r = \int_{V_{array}(in)}^{V_{array}(out)} p(V_{array})P(V_{array})/P_r dV_{array}]$$

is negligible even if equation D-8 is applied out to the limits of these extended array effective cut-in and rated speeds.